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Self inductance and mutual inductance:

The self-inductance of a circuit is used to describe the reaction of the circuit to a changing current in the circuit, while the mutual inductance with respect to a second circuit describes the reaction to a changing current in the second circuit. When a current i_1 flows in circuit 1, i_1 produces a magnetic field $\boldsymbol{B}_1$; the magnetic flux through circuit 1 due to current i_1 is Φ_{11}. Since $\boldsymbol{B}_1$ is proportional to i_1, Φ_{11} is as well. The constant of proportionality is the self-inductance L_1 of the circuit. It is defined by the equation

$$\Phi_{11} = L_1 i_1. \qquad (2)$$

The units of inductance are henrys. If a second circuit is present, some of the field $\boldsymbol{B}_1$ will pass through circuit 2 and there will be a magnetic flux Φ_{21} in circuit 2 due to the current i_1. The mutual inductance M_{21} is given by

$$\Phi_{21} = M_{21} i_1. \qquad (3)$$

The magnetic flux in circuit 1 due to a current in circuit 2 is given by $\Phi_{12} = M_{12} i_2$. An important property of the mutual inductance is that $M_{21} = M_{12}$. It is therefore sufficient to use the label M without subscripts for the mutual inductance of two circuits.
The value of the mutual inductance of two circuits can range from $+\sqrt{L_1 L_2}$ to $-\sqrt{L_1 L_2}$, depending on the flux linkage between the circuits. If the two circuits are very far apart or if the field of one circuit provides no magnetic flux through

the other circuit, the mutual inductance is zero. The maximum possible value of the mutual inductance of two circuits is approached as the two circuits produce ***B*** fields with increasingly similar spatial configurations.

If the rate of change with respect to time is taken for the terms on both sides of equation (2), the result is
$d\Phi_{11}/dt = L_1 di_1/dt$
According to Faraday’s law, $d\Phi_{11}/dt$ is the negative of the induced electromotive force. The result is the equation frequently used for a single inductor in an AC circuit—i.e.

$$\text{emf} = -L\frac{di}{dt}. \qquad (4)$$

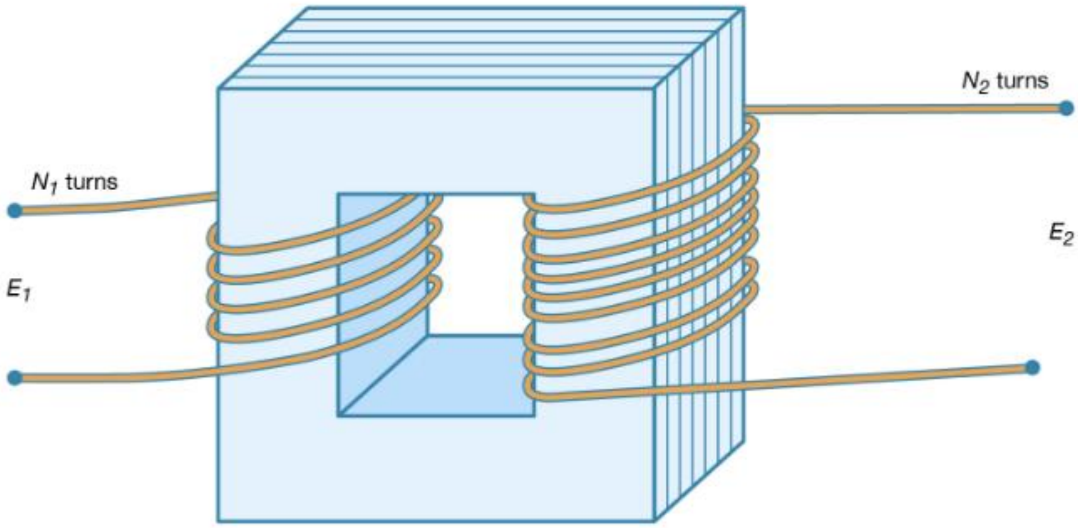
The phenomenon of self-induction was first recognized by the American scientist Joseph Henry. He was able to generate large and spectacular electric arcs by interrupting the current in a large copper coil with many turns. While a steady current is flowing in a coil, the energy in the magnetic field is given by $^{1}/_{2}Li^{2}$. If both the inductance L and the current i are large, the amount of energy is also large. If the current is interrupted, as, for example, by opening a knife-blade switch, the current and therefore the magnetic flux through the coil drop quickly. Equation (4) describes the resulting electromotive force induced in the coil, and a large potential difference is developed between the two poles of the switch. The energy stored in the magnetic field of the coil is dissipated as heat and radiation in an electric arc across the space between the terminals of the switch. Due to advances in superconducting wires for electromagnets, it is possible to use large magnets with magnetic fields of several teslas for temporarily storing electric energy as energy in the magnetic field. This is done to accommodate short-term fluctuations in the consumption of electric power.

$$\text{emf} = -L\frac{di}{dt}. \qquad (4)$$

<u>**Transformer:**</u>

A transformer is an example of a device that uses circuits with maximum mutual induction. Figure 5 illustrates the configuration of a typical transformer.

Here, coils of insulated conducting wire are wound around a ring of iron constructed of thin isolated laminations or sheets. The laminations minimize eddy currents in the iron. Eddy currents are circulatory currents induced in the metal by the changing magnetic field. These currents produce an undesirable by-product—heat in the iron. Energy loss in a transformer can be reduced by using thinner laminations, very “soft” (low-carbon) iron and wire with a larger cross section, or by winding the primary and secondary circuits with conductors that have very low resistance. Unfortunately, reducing the heat loss increases the cost of transformers. Transformers used to transmit and distribute power are commonly 98 to 99 percent efficient. While eddy currents are a problem in transformers, they are useful for heating objects in a vacuum. Eddy currents are induced in the object to be heated by surrounding a relatively nonconducting vacuum enclosure with a coil carrying a high-frequency alternating current.



In a transformer, the iron ensures that nearly all the lines of ***B*** passing through one circuit also pass through the second circuit and that, in fact, essentially all the magnetic flux is confined to the iron. Each turn of the conducting coils has the same magnetic flux; thus, the total flux for each coil is proportional to the number of turns in the coil. As a result, if a source of sinusoidally varying electromotive force is connected to one coil, the electromotive force in the second coil is given by

$$\text{emf}_2 = \text{emf}_1 \frac{N_2}{N_1}. \qquad (5)$$

Thus, depending on the ratio of N_2 to N_1 (where N_1 and N_2 are the number of turns in the first and second coils, respectively), the transformer can be either a step-up or a step-down device for alternating voltages. For many reasons, including safety, generation and consumption of electric power occur at relatively low voltages. Step-up transformers are used to obtain high voltages before electric power is transmitted, since for a given amount of power, the current in the transmission lines is much smaller. This minimizes energy lost by resistive heating of the conductors.

Faraday's law constitutes the basis for the power industry and for the transformation of mechanical energy into electric energy. In 1821, a decade before his discovery of magnetic induction, Faraday conducted experiments with electric wires rotating around compass needles. This earlier work, in which a wire carrying a current rotated around a magnetized needle and a magnetic needle was made to rotate around a wire carrying an electric current, provided the groundwork for the development of the electric motor.

Maxwell's Equation:

Maxwell's four field equations represent the pinnacle of classical electromagnetic theory. Subsequent developments in the theory have been concerned either with the relationship between electromagnetism and the atomic structure of matter or with the practical and theoretical consequences of Maxwell's equations. His formulation has withstood the revolutions of relativity and quantum mechanics. His equations are appropriate for distances as small as 10^{-10} centimetres—100 times smaller than the size of an atom. The fusion of electromagnetic theory and quantum theory, known as quantum electrodynamics, is required only for smaller distances.

Maxwell's prediction that a changing electric field generates a magnetic field was a masterstroke of pure theory. The Maxwell equations for the electromagnetic field unified all that was hitherto known about electricity and magnetism and predicted the existence of an electromagnetic phenomenon that can travel as waves with the velocity of $1/\sqrt{\varepsilon_0\mu_0}$ in a vacuum. That velocity, which is based on constants obtained from purely electric measurements, corresponds to the speed of light. Consequently, Maxwell concluded that light itself was an electromagnetic phenomenon.

Later, Einstein's special relativity theory postulated that the value of the speed of light is independent of the motion of the source of the light. Since then, the speed of light has been measured with increasing accuracy. In 1983 it was defined to be exactly 299,792,458 metres per second. Together with the cesium clock, which has been used to define the second, the speed of light serves as the new standard for length.

The circuit in Figure below is an example of a magnetic field generated by a changing electric field. A capacitor with parallel plates is charged at a constant rate by a steady current flowing through the long, straight leads in Figure.

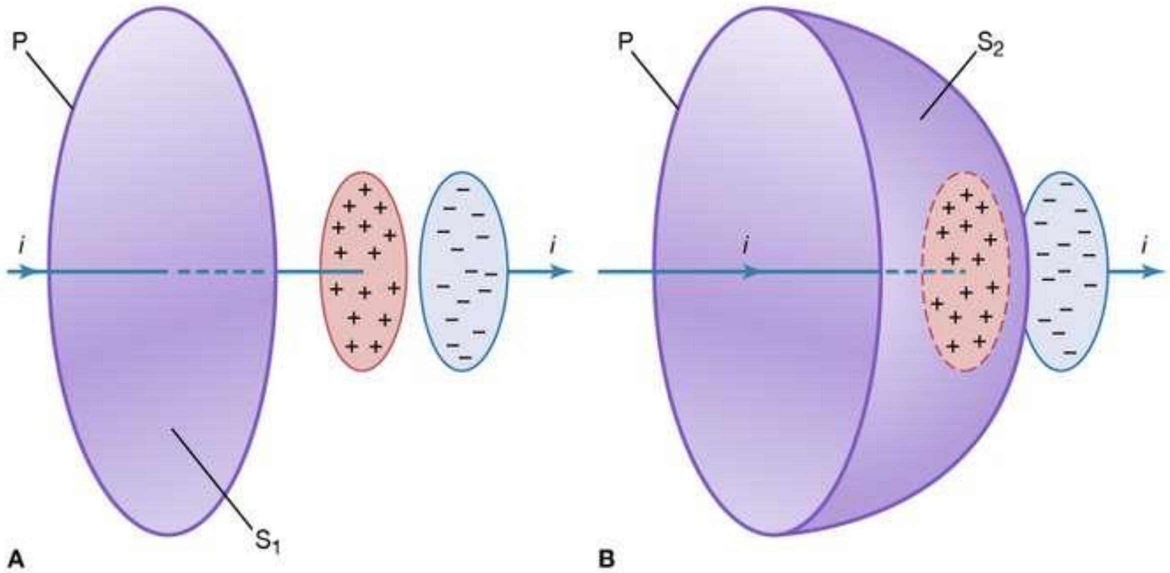


Figure: Current *i* charging a capacitor as an illustration of Maxwell's displacement current

The objective is to apply Ampère's circuital law for magnetic fields to the path P, which goes around the wire in Figure. This law (named in honour of the French physicist André-Marie Ampère) can be derived from the Biot and Savart equation for the magnetic field produced by a current.

Using vector calculus notation, Ampère's law states that the integral $\oint \boldsymbol{B} \cdot \boldsymbol{dl}$ along a closed path surrounding the current i is equal to $\mu_0 i$. (An integral is essentially a sum, and, in this case, $\oint \boldsymbol{B} \cdot \boldsymbol{dl}$ is the sum of $B \cos \theta dl$ taken for a small length of the path until the complete loop is included. At each segment of the path dl, θ is the angle between the field $\boldsymbol{B}$ and $\boldsymbol{dl}$).

The current i in Ampère's law is the total flux of the current density $\boldsymbol{J}$ through any surface surrounded by the closed path. The closed path is labeled P, and a surface S_1 is surrounded by path P. All the current density through S_1 lies within the conducting wire. The total flux of the current density is the current i flowing through the wire. The result for surface S_1 reflects the value of the magnetic field around the wire in the region of the path P. The path P is the same but the surface S_2 passes between the two plates of the capacitor. The value of the total flux of the current density through the surface should also be *i*. There is, however, clearly no motion of charge at all through the surface S_2. The dilemma is that the value of the integral $\oint \boldsymbol{B} \cdot d\boldsymbol{l}$ for the path P cannot be both $\mu_0 i$ and zero.

Maxwell's resolution of this dilemma was his conclusion that there must be some other kind of current density, called the displacement current $\boldsymbol{J}d$, for which the total flux through the surface S_2 would be the same as the current *i* through the surface S_1. $\boldsymbol{J}d$ would take, for the surface S_2, the place of the current density $\boldsymbol{J}$ associated with the movement of charge, since $\boldsymbol{J}$ is clearly zero due to the lack of charges between the plates of the capacitor. What happens between the plates while the current *i* is flowing? Because the amount of charge on the capacitor increases with time, the electric field between the plates increases with time too. If the current stops, there is an electric field between the plates as long as the plates are charged, but there is no magnetic field around the wire. Maxwell decided that the new type of current density was associated with the changing of the electric field. He found that

$$\boldsymbol{J}_d = \frac{d\boldsymbol{D}}{dt}. \qquad (6)$$

where $\boldsymbol{D} = \varepsilon_0 \boldsymbol{E}$ and $\boldsymbol{E}$ is the electric field between the plates. In situations where matter is present, the field $\boldsymbol{D}$ in equation (6) is modified to include polarization effects; the result is $\boldsymbol{D} = \varepsilon_0 \boldsymbol{E} + \boldsymbol{P}$. The field $\boldsymbol{D}$ is measured in coulombs per square metre. Adding the displacement current to Ampère's law represented Maxwell's prediction that a changing electric field also could be a source of the magnetic field $\boldsymbol{B}$. Following Maxwell's predictions of electromagnetic waves, the German physicist Heinrich Hertz initiated the era of radio communications in 1887 by generating and detecting electromagnetic waves.

$$\boldsymbol{J}_d = \frac{d\boldsymbol{D}}{dt}. \qquad (6)$$

Using vector calculus notation, the four equations of Maxwell's theory of electromagnetism are

$$\text{I. } \operatorname{div} \boldsymbol{D} = \rho, \qquad (7)$$

$$\text{II. } \operatorname{div} \boldsymbol{B} = 0, \qquad (8)$$

$$\text{III. } \boldsymbol{curl\ E} = -\frac{d\boldsymbol{B}}{dt}, \qquad (9)$$

$$\text{IV. } \boldsymbol{curl\ H} = \boldsymbol{J} + \frac{d\boldsymbol{D}}{dt}, \qquad (10)$$

where $\boldsymbol{D} = \varepsilon_0\boldsymbol{E} + \boldsymbol{P}$, and $\boldsymbol{H} = \boldsymbol{B}/\mu_0 - \boldsymbol{M}$. The first equation is based on Coulomb’s inverse square law for the force between two charges; it is a form of Gauss’s law, which relates the flux of the electric field through a closed surface to the total charge enclosed by the surface. The second equation is based on the fact that apparently no magnetic monopoles exist in nature; if they did, they would be point sources of magnetic field. The third is a statement of Faraday’s law of magnetic induction, which reveals that a changing magnetic field generates an electric field. The fourth is Ampère’s law as extended by Maxwell to include the displacement current discussed above; it associates a magnetic field to a changing electric field to a changing electric field as well as to an electric current.