DETAILS OF THE TOPIC

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DIELECTRIC PROPERTIES OF MATTER

An insulator, or alternatively known as dielectric, is a material in which the electrons are tightly bound to the nucleus of the atom; so that there are no free electrons or the number of such electrons is too low. An ideal dielectric is a material in which there is no free charge at all. Nevertheless, every material medium is composed of molecules, which in turn contain positive and negative charges entities (atomic nuclei and electrons) and in fact an equal quantity of each kind, if the dielectric is electrically neutral. However, the presence of the electric field may change the behaviour of dielectric because of its molecular structure. If the change in behaviour of the dielectric is independent of the direction of the applied field, the dielectric is termed as isotropic while if the change in behaviour of the dielectric depends on the field direction, the dielectric is anisotropic.

The change in the behaviour of the dielectric is due to the fact that the applied electric field exerts a force on each charged particle, positive particles being pushed in the direction of the field and negative particles oppositely, so that positive and negative parts of each molecule are displaced from their equilibrium position in opposite directions. ***The relative displacement of the charges is called polarization and the dielectric is said to be polarised and acts like an electric dipole.***

The molecules of a dielectric may be classified into two categories:

1. Polar molecules
2. Non-polar molecules

POLAR AND NON-POLAR MOLECULES

A system of two charges, $+q$ and $-q$ separated by a certain distance l, is called **electric dipole**. Example, HCl, $NaCl$, etc.

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Fig. 1. Electric dipole

The electric dipole moment of an electric dipole is defined as

$$\vec{p} = q\vec{l}$$

The dipole moment is a vector quantity and its direction is towards positive charge from negative charge.

The molecules having dipole moment greater than zero are called polar molecules. Water molecule has a triangular structure: two $H - O$ bonds have dipole moments $\vec{p}_1$ as shown in the Fig. 2, hence the resultant dipole moment is $\vec{p}$ is directed from O towards $H - H$ base. Thus H_2O molecule is a polar molecule.

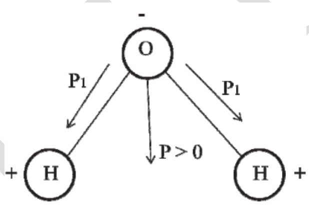

Fig. 2. Polar molecule H_2O

The molecules having zero dipole moment are called non-polar molecules. CO_2 has a linear symmetrical structure, hence the resultant dipole moment is zero. Thus, CO_2 is a polar molecule.

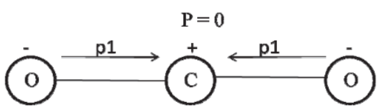

Fig. 3. Non-polar molecule

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DIELECTRIC POLARIZATION AND POLARIZABILITY

A polarised dielectric, even though it is electrically neutral on the averae produces an electric field, both at theexterior points and inside the dieletric as well. The polarization of the dielectric thus depends on the total electric field in the medium, but a part of the electric field is produced by the dielectric itself. If the average electric field acting on a molecule is $\vec{E}$ (which is in general different from the applied field) the average dipole moment $\vec{p}$ induced in each molecule of the dielctric is proportional to $\vec{E}$ provided $\vec{E}$ is not too great, *i.e.*

$$\vec{p} \propto \vec{E}$$

i.e.

$$\boxed{\vec{p} = \alpha\vec{E}} \qquad \ldots(1)$$

Where α is a constant at constant temperature and frequency by depends on the dielctric and is known as the molecular polarizability.

Equation (1) is strictly true only if $\vec{E}$ is small. When $\vec{E}$ becomes large, the rate of increase of $\vec{p}$ with $\vec{E}$ decreases. The effect is known as saturation. Saturation effects only become effective in fields of the order $10^7\ volt/m$.

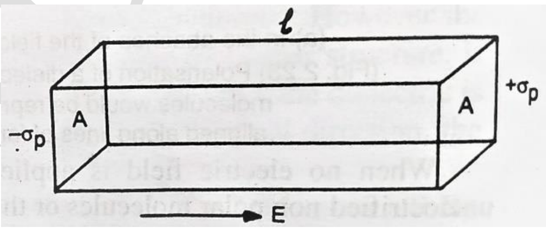

Fig. 4. Rectangular block

If there are n molecules per unit volume of the dielctric, then the total induced electric moment per unit volume $= n\alpha\vec{E}$, assuming the induced dipoles produced are all the same and that they are independent of one another. The total induced electric dipole moment per unit volume is called the polarization and is represented by the vector $\vec{P}$. Thus polarization,

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$$\vec{P} = n\alpha\vec{E} \quad \ldots(2)$$

Consider a rectangular block of the polarised dielectric of length l and uniform cross-section A. Let the surface charge densities of fictitious charge appearing at the end faces be $+\sigma_p$ and $-\sigma_p$. Therefore, charge induced on each face $= \sigma_p A$.

Therefore, induced electric dipole moment $(= ql) = \sigma_p Al$ …(3)

As $\vec{P}$ is the polarization *i.e.* electric dipole moment per unit volume and (Al) is the volume of the block, therefore the total dipole moment $= PAl$.

Equation (2) and (3), we get $PAl = \sigma_p Al$

Therefore,

$$P = \sigma_p \quad \ldots(4)$$

Thus polarization may also be defined as the surface density of charge appearing at faces perpendicular to the direction of the applied field.

DIELECTRIC CONSTANT (RELATIVE PERMITTIVITY) AND DISPLACEMENT VECTOR

We have seen that when a dielectric is placed, between parallel plates having real charges $+q$ and $-q$, then bound charges appear on the surfaces of the dielctric perpendicular to the field. These bound charges are opposite to the real charges present on the plates. Thus the effect of a dielectric is less than that in free space.

The relative permittivity (or dielectric constant) may be defined as the ratio of the electric field in free space to that in dielectric and is represented by the symbol K_e(or ϵ_r).

$$\epsilon_r = \frac{\text{Field in free space}}{\text{Field in dielctric}} = \frac{\vec{E}_0}{\vec{E}} \quad \ldots(1)$$

If $\in_0$ and $\in$ are the absolute permittivities of free space and dielctric respectively, then

$$\vec{E}_0 = \frac{1}{4\pi\epsilon_0}\frac{q\,\vec{r}}{r^3} \quad \text{and} \quad \vec{E} = \frac{1}{4\pi\epsilon_0}\frac{q\,\vec{r}}{r^3} \quad \ldots(2)$$

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Substituting these values in equation (1), we get

$$K_e = \frac{\epsilon}{\epsilon_0} \qquad \ldots(3)$$

***i.e.* relative permittivity (or dielectric constant) may be defined as the ratio of the absolute permittivity of the dielectric to that of the free space**.

From equation (2) it is clear that the electric field strength depends upon the medium. Therefore, let us introduce a new electric field vector $\vec{D}$, called **the electric displacement** or electric induction and defined in such a way that it depends only on the distribution and magnitude of charges which produce electric field but is independent of the nature of the medium. Thus, the electric displacement $\vec{D}$ due to a single point charge q at a point $\vec{r}$ is given by

$$\vec{D} = \frac{1}{4\pi}\frac{q\,\vec{r}}{r^3} \qquad \ldots(4)$$

Equation (4) may be written as $\vec{D} = \epsilon\left(\frac{1}{4\pi\epsilon}\frac{q\,\vec{r}}{r^3}\right)$ ...(5)

Using equation (2), we may write $\vec{D} = \epsilon\vec{E}$ ...(6)

Thus, it is obvious that

$$\vec{D} = \begin{cases} \epsilon_0\vec{E} & \text{(in free space)} \\ \epsilon\vec{E} & \text{(in a dielectric of absolute permittivity } \epsilon\text{)} \end{cases}$$

and the vectors $\vec{D}$ and $\vec{E}$ have the same direction.

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EXTERNAL FIELD OF A DIELECTRIC MEDIUM

Let us consider a finite piece of polarised dielctric which is characterised at each point $\vec{r}'$ by a polarization $\vec{P}(\vec{r}')$. The polarization gives rise to an electric field and we want to find this field at point $\vec{r}$ outside the dielectric body. It is convenient to calculate first the electric potential $\phi(\vec{r})$ and then obtain electric field $\vec{E}$ by the relation $\vec{E} = -grad\ \phi$.

Let the volume of the dielectric be divided into small elementary volumes. Let $\Delta V'$ be the volume element of the dielectric medium characterised by the dipole moment $\Delta\vec{p}$. Then by definition of polrization $\vec{P}(\vec{r}') = \frac{\Delta\vec{p}}{\Delta V'}$, we get

$$\Delta\vec{p} = \vec{P}(\vec{r}')\Delta V' \qquad \dots(1)$$

Therefore, the potential at point $\vec{r}$ due to this small volume element is

$$\Delta\phi(\vec{r}) = \frac{1}{4\pi\epsilon_0}\frac{\Delta\vec{p}\cdot(\vec{r}-\vec{r}')}{|\vec{r}-\vec{r}'|^3}$$

$$\Rightarrow \qquad \Delta\phi(\vec{r}) = \frac{1}{4\pi\epsilon_0}\frac{\vec{P}(\vec{r}')\cdot(\vec{r}-\vec{r}')}{|\vec{r}-\vec{r}'|^3}\Delta V' \qquad \dots(2)$$

[by using equation (1).]

where $(\vec{r}-\vec{r}')$ is a vector from volume element $\Delta V'$ to the point $A(x, y, z)$ under consideration.

The magnitude of $(\vec{r}-\vec{r}')$ is given by

$$|\vec{r}-\vec{r}'| = \sqrt{(x-x')^2 + (y-y')^2 + (z-z')^2} \qquad \dots(3)$$

The net potential at point $\vec{r}$ is obtained by summing up the contribution from all parts of the dielectric *i.e.*

$$\phi(\vec{r}) = \sum \frac{1}{4\pi\epsilon_0}\frac{\vec{P}(\vec{r}')\cdot(\vec{r}-\vec{r}')}{|\vec{r}-\vec{r}'|^3}\Delta V' \qquad \dots(4)$$

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If the volume $\Delta V'$ is infintesimal, we write dV' for $\Delta V'$ and replace summation by integration *i.e.*

$$\phi(\vec{r}) = \frac{1}{4\pi\epsilon_0}\int_V \frac{\vec{P}(\vec{r}')\cdot(\vec{r}-\vec{r}')}{|\vec{r}-\vec{r}'|^3}dV' \qquad \ldots(5)$$

From this result $\phi(\vec{r})$ may be evaluated diretly if the function form of $\vec{P}$ is known. It is sometimes useful to express equation (5) in a rather different way by means of simple mathematical transformations.

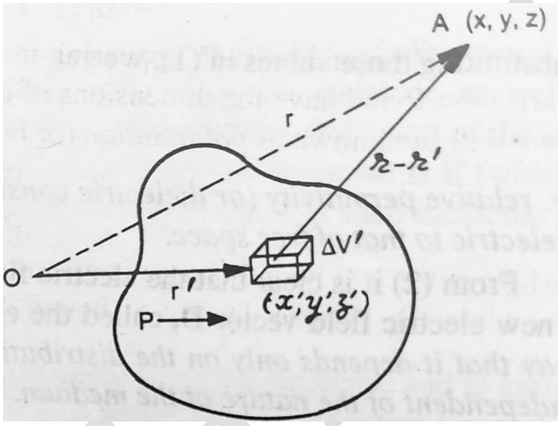

Fig. 5. External field of a dielectric medium

If ∇' operator involves with respect to the primed coordinates, then

$$\nabla'\left(\frac{1}{|\vec{r}-\vec{r}'|}\right) = \frac{\vec{r}-\vec{r}'}{|\vec{r}-\vec{r}'|^3} \qquad \ldots(6)$$

It may be seen by direct application of the gradient operator in cartesian coordinates. Using equation (6) we may write

$$\frac{\vec{P}(\vec{r}')\cdot(\vec{r}-\vec{r}')}{|\vec{r}-\vec{r}'|^3} = \nabla'\left(\frac{1}{|\vec{r}-\vec{r}'|}\right) \qquad \ldots(7)$$

This equation may be further transformed by means of following vector identity

$$div'\left(S\vec{A}\right) = S\, div'\vec{A} + \vec{A}\cdot\nabla' S \qquad \ldots(8)$$

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Where S is any scalar point function and $\vec{A}$ is an arbitrary vector point function. Here again prime indicates the differentiation with respect to the primed coordinates. Using $S = \frac{1}{|\vec{r}-\vec{r}'|}$ and $\vec{A} = \vec{P}$, equation (8) yields

$$div'\left(\frac{\vec{P}}{|\vec{r}-\vec{r}'|}\right) = \frac{1}{|\vec{r}-\vec{r}'|}\, div'\vec{P} + \vec{P}\cdot\nabla'\left(\frac{1}{|\vec{r}-\vec{r}'|}\right)$$

$$\therefore \quad \vec{P}\cdot\nabla'\left(\frac{1}{|\vec{r}-\vec{r}'|}\right) = div'\left(\frac{\vec{P}}{|\vec{r}-\vec{r}'|}\right) - \frac{1}{|\vec{r}-\vec{r}'|}\, div'\vec{P} \qquad \ldots(9)$$

Comparing equations (7) and (9), we note that integrand of equation (5) takes form

$$\frac{\vec{P}(\vec{r}')\cdot(\vec{r}-\vec{r}')}{|\vec{r}-\vec{r}'|^3} = div'\left(\frac{\vec{P}}{|\vec{r}-\vec{r}'|}\right) - \frac{1}{|\vec{r}-\vec{r}'|}\, div'\vec{P} \qquad \ldots(10)$$

Hence, the potential $\phi(\vec{r})$, from equation (5) takes form

$$\phi(\vec{r}) = \frac{1}{4\pi\epsilon_0}\int_V div'\left(\frac{\vec{P}}{|\vec{r}-\vec{r}'|}\right)dV' - \frac{1}{4\pi\epsilon_0}\int_V \frac{1}{|\vec{r}-\vec{r}'|}\, div'\vec{P}dV' \qquad \ldots(11)$$

From Gauss divergence theorem, changing volume integral of $div'\left(\frac{\vec{P}}{|\vec{r}-\vec{r}'|}\right)$ into surface integral, we get

$$\int_V div'\left(\frac{\vec{P}}{|\vec{r}-\vec{r}'|}\right)dV' = \int_S \frac{\vec{P}\cdot\hat{n}}{|\vec{r}-\vec{r}'|}dS' \qquad \ldots(12)$$

Using equation (12), the potential $\phi(\vec{r})$ finally takes the form

$$\phi(\vec{r}) = \frac{1}{4\pi\epsilon_0}\int_S \frac{\vec{P}\cdot\hat{n}}{|\vec{r}-\vec{r}'|}dS' + \frac{1}{4\pi\epsilon_0}\int_V \frac{-\,div'\vec{P}}{|\vec{r}-\vec{r}'|}dV' \qquad \ldots(13)$$

The quantites $\vec{P}\cdot\hat{n}$ and $-\,div'\vec{P}$ appearing in the integral of equation (13), are two scalar functions obtained from the polarization $\vec{P}$ and have the dimesions of charge per unit area and

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charge per unit volume respectively. Therefore, they are known as polarization (or bound) volume charge density respectively and are dentoed by

$$\sigma_p = \vec{P} \cdot \hat{n} = P_n \quad \text{and} \quad \rho_p = -\, div'\vec{P} \qquad \dots(14)$$

The surface density of bound charge is given by the normal components of polarization vector normal to the surface, while the volume density of bound charge is a measure of the non-uniformity of the polarization inside the material.

Using equation (14), equation (13) for potential $\phi(\vec{r})$ takes the form

$$\phi(\vec{r}) = \frac{1}{4\pi\epsilon_0}\int_S \frac{\sigma_p}{|\vec{r}-\vec{r}'|}dS' + \frac{1}{4\pi\epsilon_0}\int_V \frac{\rho_p}{|\vec{r}-\vec{r}'|}dV' \qquad \dots(15)$$

i.e.

$$\phi(\vec{r}) = \frac{1}{4\pi\epsilon_0}\int_S \frac{dq_p{}'}{|\vec{r}-\vec{r}'|} \qquad \dots(16)$$

It is clear that the potential $\phi(\vec{r})$ arises due to a charge distribution. In other words, the dielectric material has been replaced by an appropriate polarization (or bound) charge distribution.

From equations (16), (15) and (13), it is obvious that the net polarization charge of a dielectric is

$$q_p = \int_S \sigma_p dS' + \int_V \rho_p dV' = \int_S \vec{P}\cdot\hat{n}\, dS' + \int_V \left(-\, div'\vec{P}\right)dV' \qquad \dots(17)$$

And that is must be zero since we had assumed that the dielectric as a whole is electrically neutral. This result immediately follows since q_p vanishes as a consequence of the divergence of the divergence theorem.

Now, we have two distinct expressions for electrostatis potential $\phi(\vec{r})$ due to a polarised dielectric given by equations (15) and (16). Both of them are correct: but it is found that the

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latter exression is more convenient in most cases. The electric field $\vec{E}$ is obtaine by relation $\vec{E} = -grad\ \phi$. As ϕ is a function of (x, y, z) it is desirable to use the gradient operation with respect to unprimed coordinates. Evidently, ∇' (gradient operator with respect to primed coordinates) operating on a function of $|\vec{r} - \vec{r}'|$ is equal to $-\nabla$ (gradient operator with respect to unprimed coordinates) appear only in the function $\frac{1}{|\vec{r}-\vec{r}'|}$ and noting that

$$\nabla\left(\frac{1}{|\vec{r} - \vec{r}'|}\right) = -\nabla'\left(\frac{1}{|\vec{r} - \vec{r}'|}\right)$$

And using equation (6), we get

$$\vec{E}(\vec{r}) = \frac{1}{4\pi\epsilon_0}\left[\int_S \frac{\sigma_p(\vec{r} - \vec{r}')}{|\vec{r} - \vec{r}'|^3} dS' + \frac{1}{4\pi\epsilon_0}\int_V \frac{\rho_p(\vec{r} - \vec{r}')}{|\vec{r} - \vec{r}'|^3} dV'\right] \quad \ldots(18)$$

This equation gives general expression for the electric field at an external point $\vec{r}$ due to a dielectric medium.

THE ELECTRIC FIELD IN A MATERIAL MEDIUM

Ley us now find an expression for the elctric field in a small polarised medium. We are actually interested in the macroscopic electric field *i.e.* the average electric field in a small region of the dielectric which, neverthless, contains a large number of molecules. The electric field inside a dielectric is different from the macroscopic field, sine the polarised medium of positive nuclei and electrons in which fields of millions volts/cm. exist and which may point in any direction depending on the location o fpoint under consideration. The macroscopic electric field is the force per unit charge on a test charge embedded in the dielctric in the limit where the test charge is so small that is does not itself affect the charge distribution. This test charge must be dimensioanlly small from the macroscopic point of view (which we call point charge), but it will be large compared with the size of a molecule.

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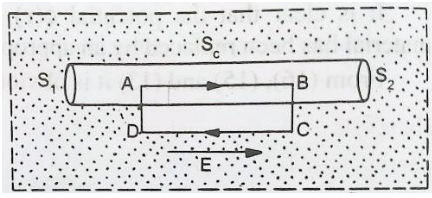

Fig. 6. Electric field in a material medium

It is very difficult to use this fundamental definition of macroscopic electric field $\vec{E}$ directly to obtain an expression for the field, since we would have to calculate the force on a charged body of extended size, and then go to the limit as the size of the object is decreased. However for this purpose we can use another property of the electric field. The electrostatic field in a dielectric must have the same basic propertiees which are applicable to electric field $\vec{E}$ in vacuum; in articular, $\vec{E}$ is a conservative field and hence derivable from a sclar potential ϕ *i.e.*

$$\vec{E} = -grad\,\phi \qquad ...(1)$$

Since $$curl\; grad\,\phi = 0$$

This implies that $$curl\,\vec{E} = 0$$

or $$\oint \vec{E} \cdot d\vec{l} = 0 \qquad ...(2)$$

Let us apply equation (2) to the path $ABCD$ as shown in Fig. 6, where the segment AB lies in needle shaped cavity cut out of the dielectric and the segment CD lies in the dieelctric proper since the segment AB and BC may be made arbitrarily small, the line integral reduces to

$$\oint \vec{E} \cdot d\vec{l} = \oint_A^B \vec{E}_v \cdot d\vec{l} + \oint_C^D \vec{E}_v \cdot d\vec{l} = 0 \qquad ...(3)$$

Where the subscripts v and d refer to vacuum and dielectric respectively. If l is the length of the path AB or CD, then equation (3) may be expressed as $E_{vt}l - E_{dt}l = 0$ where subscript t stand for tangential component. This implies

$$E_{vt} = E_{dt} \qquad \ldots(4)$$

This equation is valid for any orinetation of the needle shaped cavity. Further it the needle is oriented along the direction of $\vec{E}$ then we have $E_{dt} = E_d$ and by symmetry the field in the cavity is along the direction of the needle, *i.e.* $E_v = E_{vt}$. Thus, in this case equation (4) takes the form

$$E_v = E_d$$

i.e. the electric field in a dielectric is equal to the electric field inside a needle shaped cavity in the dielectric provided the cavity axis is oriented to the direction of the electric field.

This statement is strictly true only for isotropic dielectric; because for anisotropic dielectrics the symmetry argument fails.

The problem of calculating the electric field inside a dielectric, thus reduces to caculating th electric field inside a needle shaped cavity in the dielectric. But the electric field in the cavity is an external field and hence may be determined by means of results described in last section. Just as we discussed earlier, we assume here that the polarization of the dielectric is a given function $P(x', y', z')$ and calcuate the potential and electric field arising from this polarization. Taking the field point $\vec{r}$ at the center of the cavity, the potential from equation (15) of last section is given by

$$\phi(\vec{r}) = \frac{1}{4\pi\epsilon_0}\left\{\int_{S'+S} \frac{\sigma_p(x', y', z')}{|\vec{r} - \vec{r}'|} dS' + \int_{V-V\prime} \frac{\rho_p(x', y', z')}{|\vec{r} - \vec{r}'|} dV'\right\} \qquad \ldots(5)$$

Where $V - V'$ is the volume of the dielctric excluding the needle, S_0 is the exterior surface of the dielectric and $S' = S_1 + S_2 + S_c$ are the needle surfaces. But from Fig. 6, $\sigma_p = 0$ on the cylindrical surface S_c of the needle may be assumed arbitraruly thin to make the surfacce S_1 and S_2 to possess negligibly small area; so that only the exterior surfaces of the dielctric contribute to the surface integral. Sonsequently the surface integral of equation (5) becomes

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identical in form to the surface integral of equation (15) of last section. The volume integral of equation (5) excludes the cavity; but it may be seen that the contribution of the cavity to the volume integral is negligible if the cavity is arbitrarily thin. Thus, it is not necessary to exclude the volume V' and hence equation (5) becomes similar in form to equation (15) of last section. In other words equaton (15) of last section gives the potential $\phi(\vec{r})$ irrespective of whether the point $\vec{r}$ is located inside or outside the dielectric.

The electric field $\vec{E}$ at $\vec{r}$ may be calculated by the relation $\vec{E}(\vec{r}) = -grad\ \phi(\vec{r})$. But this differs only by a negligible amount from equation(18) of last section. Thus, equation (18) of last section, *i.e.*

$$\vec{E}(\vec{r}) = \frac{1}{4\pi\epsilon_0}\left[\int_S \frac{\sigma_p(\vec{r}-\vec{r}')}{|\vec{r}-\vec{r}'|^3}dS' + \frac{1}{4\pi\epsilon_0}\int_V \frac{\rho_p(\vec{r}-\vec{r}')}{|\vec{r}-\vec{r}'|^3}dV'\right] \quad \ldots(6)$$

gives the contribution of the dielectric medium to the electric field $\vec{E}(\vec{r})$, independently of whether the point $\vec{r}$ lies inside or outside the medium.

THE GAUSS THEOREM IN A DIELECTRIC : THE ELECTRIC DISPACEMENT

The Gauss theorem relates the electric flux and charge. The theorem states that **'the net electric flux across an arbitrary closed surface drawn in an electric field is equal to $\frac{1}{\epsilon_0}$ times the total charge enclosed by the surface'**. Now, we want to extend this theorem for a region containing free charge embedded in a dielectric.

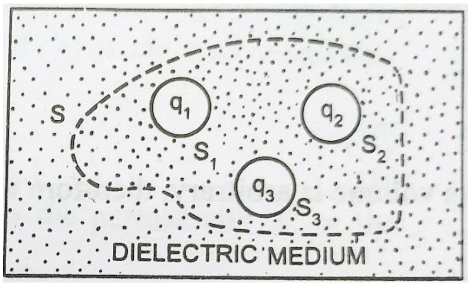

Fig. 7. Imaginary closed surface in a dielectric medium

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In Fig. 7, the dotted surface S is an arbitrary closed surface drwan in a dielectric medium. There is certain amount of free charge Q in the volume bounded by surface S. Let us assume that this free charge exists on the surfaces of three conductors in amounts q_1, q_2, q_3. In a dielectric there also exists certain amount of polarization (bound) charge Q_p. Hence, by Gauss's theorem

$$\oint \vec{E} \cdot \hat{n}\, dS = \frac{1}{\epsilon_0}\left(Q + Q_p\right) \qquad \ldots(1)$$

where $Q = q_1 + q_2 + q_3$, is the total charge and Q_p is the polarization (bound) charge given by

$$Q_p = \int_{S_1+S_2+S_3} \vec{P} \cdot \hat{n}\, dS + \int_V \left(-\,div\vec{P}\right) dV \qquad \ldots(2a)$$

Here V is the volume of the dielectric enclosed by S. As there is no boundary of the dielectric at S, therefore the surface integral in equation (2a) does not contain a contribution from S. If we transform volume integral in equation (2a) into surface integral by means of Gauss divergence theorem, we must include contributions from all surface bounding V, namely S_1, S_2, S_3 *i.e.*

$$\int_V \left(-\,div\vec{P}\right) dV = -\left[\int_{S_1+S_2+S_3} \left(\vec{P} \cdot \hat{n}\right) dS + \oint_S \left(\vec{P} \cdot \hat{n}\right) dS\right]$$

Using above equation, we note that

$$Q_p = \oint_S \left(\vec{P} \cdot \hat{n}\right) dS \qquad \ldots(2b)$$

Substituting this value in equation (1), we get

$$\oint_S \vec{E} \cdot \hat{n}\, dS = \frac{Q}{\epsilon_0} - \frac{1}{\epsilon_0}\oint_S \left(\vec{P} \cdot \hat{n}\right) dS$$

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i.e. $$\oint_S \left(\vec{E} + \frac{\vec{P}}{\epsilon_0}\right) \cdot \hat{n}\, dS = \frac{Q}{\epsilon_0}$$

Multiplying throughout by ϵ_0, we get

$$\oint_S \left(\epsilon_0\vec{E} + \vec{P}\right) \cdot \hat{n}\, dS = Q \qquad \ldots(3)$$

This equation states that the flux of the vector $\left(\epsilon_0\vec{E} + \vec{P}\right)$ through a closed surface is equal to the total free charge enclosed by the surface. This vector quantity is named as electric displacement vector $\vec{D}$, *i.e.*

$$\boxed{\vec{D} = \epsilon_0\vec{E} + \vec{P}} \qquad \ldots(4)$$

Evidently, the electric displacement vector $\vec{D}$ has the same units as $\vec{P}$ *i.e.* charge per unit area.

In terms of electric displacement vector $\vec{D}$, equation (3) becomes

$$\boxed{\oint_S \vec{D} \cdot \hat{n}\, dS = Q} \qquad \ldots(5)$$

i.e. **the flux of electric displacement vector $\vec{D}$ across an arbitrary closed surface is equal to the total free charge enclosed by the surface**. This result is usually referred to as **'Gauss's theorem in a dielectric'**.

ELECTRIC SUSCEPTIBILITY OF DIELECTRIC MATERIALS

The phenomenon of electric polarization occurs when a dielectric is placed in an electric field. Accordingly, the three electric vectors $\vec{E}$, $\vec{P}$ and $\vec{D}$ are related as

$$\vec{D} = \epsilon_0\vec{E} + \vec{P} \qquad \ldots(1)$$

But $$\vec{D} = \epsilon\vec{E} = K_e\epsilon_0\vec{E}$$

Therefore, $$\vec{P} = K_e\epsilon_0\vec{E} - \epsilon_0\vec{E}$$

i.e. $$\vec{P} = \epsilon_0 (K_e - 1)\vec{E} \qquad \ldots(2)$$

where ϵ_0 is the permittivity of free space and is constant while K_e is constant for a medium (Here, it is known as dielectric constant).

The term $(K_e - 1)$ is replaced by another constant χ_e, so that

$$\vec{P} = \epsilon_0 \chi_e \vec{E} \qquad \ldots(3)$$

i.e. $$\chi_e = \frac{\vec{P}}{\epsilon_0 \vec{E}} \qquad \ldots(4)$$

χ_e is called the electric susceptibility of dielectric and is characteristics of the material. The electric susceptibility of the dielectric may be defined as the ratio of polarization to the product of electric field intensity in the dielectric and free space permittivity. The dielectric susceptibility is dimensionless.

Three quantities ϵ, K_e and χ_e are simply different ways of describing the properties of the dielectric.

From equations (2) and (3), we find

$$\boxed{\chi_e = (K_e - 1)}$$

$$\Rightarrow \quad \boxed{K_e = 1 + \chi_e} \qquad \ldots(5)$$

This is the relation between dielectric constant and susceptibility.

RELATION BETWEEN $\vec{E}$, $\vec{D}$ and $\vec{P}$

Let us derive the relation between polarization vector ($\vec{P}$), displacement ($\vec{D}$) and electric field ($\vec{E}$).

When a dielectric placed in the parallel plates of a capacitor, which is subjected to an external electric field $\vec{E}_0$, then dielectric is polarised. The charges are induced at the surfaces of the dielectric. If q_0 is the charge on the plates of capacitor, and q' is the induced charge on the boundary of dielectric, the resultant field at any point in the dielectric is given as

$$\vec{E} = \vec{E}_0 - \vec{E}_p \qquad \ldots(1)$$

where $\vec{E}_0 = \frac{q_0}{4\pi A}$ and $\vec{E}_p = \frac{q'}{4\pi A}$, A is the area of dielectric slab (in $\vec{E}_p$, ϵ_0 has been used, as there is no free charge inside the polarised dielectric). Thus,

$$\vec{E} = \frac{q_0}{4\pi A} - \frac{q'}{4\pi A} \qquad \ldots(2)$$

But $\frac{q'}{A}$ is the surface density of the induced charges, called the electric polarization $\vec{P}$, therefore

$$\epsilon_0\vec{E} = \frac{q_0}{A} - \vec{P}$$

i.e.
$$\frac{q_0}{A} = \epsilon_0\vec{E} + \vec{P} \qquad \ldots(3)$$

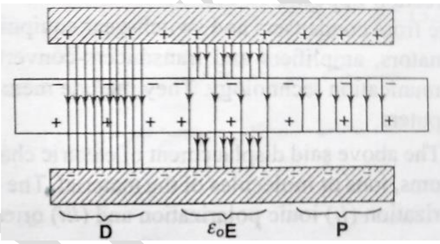

Fig. 8. Representation of $\vec{D}$, $\vec{E}$ and $\vec{P}$ in the dielectric placed between parallel plates of capacitor

The quantity on the left side is the free surface charge density and called electric displacement vector $\vec{D}$ (name introduced by Maxwell) and measured in $Coulomb/m^2$.

Thus,
$$\boxed{\vec{D} = \epsilon_0\vec{E} + \vec{P}} \qquad \ldots(4)$$

where $\vec{D}$ is the electric displacement vector or the flux density and is defined as the number of lines of force received by a unit area.

The equation (4) represents the relation between $\vec{E}$, $\vec{D}$ and $\vec{P}$.

In free space, when there is no dielectric, $\vec{P} = 0$, we get

$$\boxed{\vec{D} = \epsilon_0\vec{E}} \qquad \ldots(5)$$

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PROBLEM SET – DIELECTRIC PROPERTIES OF MATTER

MULTIPLE CHOICE QUESTIONS

1. Net outflow of flux through a closed surface enclosing charge q_0 is

   (a) q_0 (b) $1/q_0$ (c) $q_0\varepsilon_0$ (d) q_0/ε_0

2. The flux density is related to the electric filed as

   (a) $\vec{D} = \varepsilon + \vec{E}$ (b) $\vec{D} = \varepsilon - \vec{E}$ (c) $\vec{D} = \varepsilon/\vec{E}$ (d) $\vec{D} = \varepsilon\vec{E}$

3. The polarization $\vec{P}$ in a dielectric is related to electric field $\vec{E}$ and the electric flux density $\vec{D}$ by the relation

   (a) $\vec{E} = \varepsilon_0\vec{D} + \vec{P}$ (b) $\vec{D} = \varepsilon_0\vec{P} + \vec{E}$

   (c) $\vec{D} = \varepsilon_0\vec{E} + \vec{P}$ (d) $\vec{D} = \varepsilon_0(\vec{E} + \vec{P})$

4. The dielectric constant K_e and electric susceptibility χ_e are related as

   (a) $K_e = 1 - \chi_e$ (b) $K_e = 1 + \chi_e$ (c) $K_e = \varepsilon_0\chi_e$ (d) $K_e = 1 + \varepsilon_0\chi_e$

5. The dipole moment per unit volume of a solid is the sum of all the individual dipole moments and is called

   (a) Polarization (b) permittivity

   (c) electrostatic moment (d) none of these

6. The net dipole moment of CO_2 molecule (CO_2 molecule is symmetric linear molecule) is

   (a) $2ql$ (b) ql (c) 0 (d) none of these

7. For a dielectric, the value of dielectric constant is 1.329. The electric susceptibility (χ_e) of the dielectric is

   (a) 1.329 (b) 0.329 (c) 2.329 (d) -0.329

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8. For a gas the value of dielectric constant at 0^oC is 1.000038. The electric susceptibility at this temperature is

   (a) 1.000038 (b) 0.00038 (c) 0.000038 (d) 2.000038

SHORT QUESTIONS

1. What do you understand by dipole moment and polarization?
2. What is a dielectric?
3. What are polar and non-polar molecules? Give examples.
4. Write the relation between flux density and intensity of the electric field?
5. State Gauss law in dielectric.
6. What do you understand by displacement vector?
7. Write the relation between $\vec{E}$, $\vec{P}$ and $\vec{D}$?
8. Show that $\vec{P} = \varepsilon_0(K_e - 1)\vec{E}$, where $\vec{P}$ is the polarization vector.
9. Define electric susceptibility and find its relation with dielectric constant.
10. Write note on three electric vectors $\vec{E}$, $\vec{P}$ and $\vec{D}$.

LONG QUESTIONS

1. Derive the expression of electric field in dielectric material?
2. State and prove the Gauss law in dielectrics.
3. What are the electric vectors in dielectrics? Name and find relationship between them.
4. Explain the phenomenon of polarization of dielectric medium and show that $K_e = 1 + \chi_e$, where the symbols have their usual meaning.
5. What do you understand by dielectric and dielectric polarization? Define $\vec{E}$, $\vec{P}$ and $\vec{D}$ and establish a relation between them.

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NUMERICAL QUESTIONS

1. Two parallel plates have equal and opposite charges and separated by a distance 5 mm thick having dielectric constant 3. If the field intensity in the dielectric is $10^6\ Volt/m$. Calculate (a) the polarization $\vec{P}$ in the dielectric and (b) the displacement vector $\vec{D}$ in the dielectric.
2. An isotropic material of relative permittivity K_e is placed normal to a uniform external electric field with an electric displacement vector of magnitude $5 \times 10^{-4}\ m^2$. If the volume of the slab is $0.5\ m^3$ and magnitude of polarization is $4 \times 10^{-4}\ m^2$, find the value of K_e and total dipole moment of the slab.
3. For a dielectric, the value of dielectric constant is 1.329. Calculate the electric susceptibility (χ_e) of the dielectric material.
4. For a gas the value of dielectric constant at 0^oC is 1.000038. Calculate the electric susceptibility at this temperature.