Statistical Quality Control

Semester IV **STAT CC410**

Unit 2

- ***p* and *d* – Chart for Fixed Sample size**
- ***p* and *d* – Chart for Variable Sample size**
- **Interpretation of *p*-chart**

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p and *d* – Chart for Fixed Sample size

- If sample size for each sample remains constant, i.e., $n_1 = n_2 = \ldots = n_k = n$, then an estimate of the population proportion *p* is given by,
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$$\hat{p} = \bar{p} = \frac{\sum_{i=1}^{k} np_i}{\sum_{i=1}^{k} n} = \frac{n\sum_{i=1}^{k} p_i}{nk} = \frac{1}{k}\sum_{i=1}^{k} p_i \qquad \text{-------- (4)}$$

- In this case, the same set of control limits can be used for all the samples inspected and we can use either *p*-chart or *d*-chart.

p and *d* – Chart for Variable Sample size

Method 1

- If the number of items in each sample varies, for *p*-chart separate control limits have to be computed for each sample but the central line remains the same
- Whereas for *d*-chart control limits as well as central line has to be computed for each sample. This type of limits are known as variable control limits.
- In such situation, *p*-chart is relatively simple and preferred to *d*-chart.

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Method 2

- If *n* varies separate control limits are to be calculated for each sample. Since $S.E.(p) = \sqrt{\frac{PQ}{n}}$,
- It is obvious that smaller the sample size wider the control band and vice-versa.
- If the sample size does not vary, then a single set of control limits based on the average sample size $\frac{1}{k}\sum_{i=1}^{k} n_i$ can be used.
- For practical purposes, this holds good for situations where largest sample size does not exceed the smallest sample size more than 20% of the smallest sample size.

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- Alternatively, for all sample sizes two sets of control limits, one based on the largest sample size and other based on the smallest sample size can be used.
- The largest sample size gives the smallest control band which is called the inner band and smallest sample size gives the largest control band which is called the outer band indicates the process in control.
- While the points lying outside the outer band are indicative of the presence of assignable causes of variation which must be searched and rectified.
- For other points action should be based on the exact control limits.

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Method 3

• Another procedure is to standardize the variate i.e., instead of plotting *p* or *d* on the control chart we plot corresponding standardized value, viz.,

$$Z = \frac{p - \bar{p}}{S.E.(p)} = \frac{p - \bar{p}}{\sqrt{\text{var}(p)}} = \frac{p - \bar{p}}{\sqrt{\frac{P'Q'}{n}}} = \frac{p - \bar{p}}{\sqrt{\frac{P'(1-P')}{n}}}$$

(standard given)

$$Z = \frac{p - \bar{p}}{\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}}$$

---------- (5)

(standard not given)

• This stablises our variable and the resulting chart is called stabilised *p*-chart or *d*-chart.

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Method 3

• In this case, the control limits as well as central line for *p* or *d* chart are invariant with n i.e., they are constants independent of n being given by,

$$\left.\begin{aligned} UCL &= 3 \\ LCL &= -3 \\ CL &= 0 \end{aligned}\right\} \quad \text{---------- (6)}$$

• Hence the problem of variable control limits can be solved with a little more computational work.

Interpretation of *p*-chart

1) (a) From the *p*-chart, a process is said to be in statistical control in the same way as is done in $\overline{X}$ and *R* chart.

(b) If all the sample points fall within the control limits without showing any specific pattern, the process is said to be in control.

(c) In such case, the observed variations in the fraction defective are attributed to the stable pattern of chance causes and the average fraction defective $\overline{p}$ is taken as the standard fraction defective *P*.

Interpretation of *p*-chart

2) (a) Points outside the UCL are known as high spots. These suggests deterioration in the quality and should be regularly reported to production engineers.

(b) The reasons for such deterioration could possibly be known and removed if the details of conditions under which data were collected, were known.

(c) Of particular interest and importance is, if there was any change of inspection or inspection standards.

Interpretation of *p*-chart

3) (a) Points below the LCL are called low spots. Such points represent a situation showing improvement in the product quality.

(b) However, before taking this improvement for guaranteed, it should be investigated if there was any slackness in inspection or not.

4) (a) When a number of points fall outside the control limits, a revised estimate of *P* should be obtained by eliminating all the points that fall above UCL.

(b) It is assumed that points that fall below LCL are not due to faulty inspection. The standard fraction defective *P* should be revised periodically in this way.

Note – The interpretation for the control chart for number of defects (*d-chart*) is same as that of *p*-chart.