Statistical Quality Control

Semester IV **STAT CC410**

Unit 2

- Control Chart for Number of Defects Per Unit (c-chart)
- Control Limits for c – Chart
- c-Chart for Variable Sample Size or u-Chart
- Application of c-chart

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Control Chart for Number of Defects Per Unit (c-chart)

- Before we start to discuss the theory behind *c*-chart, it is important to distinguish between defect and defective.
- An article which does not conform to one or more of the specifications is known as defective while any instance of article's lack of conformity to specifications is a defect.
- Thus every defective contains one or more of the defects, e.g., a defective casting may further be examined for blow holes, cold shuts, rough surface, weak structure, etc.
- Unlike *d* or *np* chart which applies to the number of defectives in a sample, *c*-chart applies to the number of defects per unit. Sample size for *c*-chart may be a single unit like a radio or group of units or it may be a unit of fixed time, length, area, etc.

Ex.

(i) In case of surface defects, area of the surface is the sample size.

(ii) In case of casting defects, a single part such as base plate, side cover is the sample size.

- However, sample size should be constant in the sense that different samples have essentially equal opportunity for the occurrence of defects.

Control Limits for *c* – Chart

- In many manufacturing process, the sample size *n* is very large since the opportunity of defects to occur are numerous and the probability *p* of occurrence of a defect in any one spot is very small such that *np* is finite.
- In such situations from statistical theory we know that the pattern of variations in data can be represented by Poisson distribution and consequently *3-σ* control limits based on Poisson distribution are used.
- For Poisson distribution mean and variance are equal, let us assume that *c* is the Poisson variate with parameter λ, then we get

$$E(c) = \lambda, \text{ and } Var(c) = \lambda$$

Thus, *3-σ* control limits for *c*-chart are given by,

$$\left.\begin{aligned} UCL_c &= E(c) + 3\sqrt{Var(c)} = \lambda + 3\sqrt{\lambda} \\ LCL_c &= E(c) - 3\sqrt{Var(c)} = \lambda - 3\sqrt{\lambda} \\ CL_c &= \lambda \end{aligned}\right\} \quad \text{.........} \ (1)$$

Case (i) Standards given

If λ' is the specified value of λ, then

$$\left.\begin{aligned} UCL_c &= \lambda' + 3\sqrt{\lambda'} \\ LCL_c &= \lambda' - 3\sqrt{\lambda'} \\ CL_c &= \lambda' \end{aligned}\right\} \qquad \text{......... (2)}$$

Case (ii) Standards not given

If the value of λ is not known, it is estimated by the mean number of defects on the i^{th} (i = 1,2,….k) inspected unit, then an estimate of λ is given by

$$\left.\hat{\lambda} = \bar{c} = \frac{1}{k}\sum_{i=1}^{k} c_i \right\} \qquad \text{......... (3)}$$

It can be seen that $\bar{c}$ is an unbiased estimate of λ. Then control limit in this case is given by,

… continued

$$\left.\begin{aligned} UCL_c &= \bar{c} + 3\sqrt{\bar{c}} \\ LCL_c &= \bar{c} - 3\sqrt{\bar{c}} \\ CL_c &= \bar{c} \end{aligned}\right\} \quad \ldots\ldots\ldots (4)$$

- Since *c* can not be negative, if LCL comes out to be negative, it is regarded as zero.
- The central line, UCL and LCL are drawn by the above formula.
- The observed number of defects on the inspected units are plotted on the control chart.
- The interpretation for *c*-chart are similar to those of *p*-chart.

Note – Usually k, the number of samples (inspected units) is taken from 20 to 25. Normal approximation to Poisson distribution may be used provided $\bar{c} < 5$.

c-Chart for Variable Sample Size or u-Chart

- In this case instead of plotting c, the statistic $u = c/n$ is plotted, n being the sample size which is varying.
- If n_i is the sample size and c_i is the total number of defects observed in the i^{th} sample then,

$$u_i = \frac{c_i}{n_i}, \quad (i = 1,2,\ldots,k) \qquad \text{----------} \ (1)$$

gives the average number of defects per unit for the i^{th} sample.

- In this case, an estimate of λ, the mean number of defects per unit in the lot, based on all the k-samples is given by,

$$\hat{\lambda} = \bar{u} = \frac{1}{k}\sum_{i=1}^{k} u_i \qquad \text{----------} \ (2)$$

- We know that if $\bar{X}$ is the mean of the random sample of size *n*, then

$$S.E(\bar{X}) = \sigma_x / \sqrt{n}$$

- Hence the standard error of the average number of defects per unit is given by,

$$S.E(u) = \sqrt{\lambda / n} = \sqrt{\bar{u} / n} \qquad \text{[using (2)]} \quad ---------- (3)$$

where $\lambda = \bar{u}$

- Hence, *3-σ* control limits for *u*-chart or *c*-chart for variable sample size are given by

$$\left.\begin{aligned} UCL_u &= \bar{u} + 3\sqrt{\bar{u} / n} \\ LCL_u &= \bar{u} - 3\sqrt{\bar{u} / n} \\ CL_u &= \bar{u} \end{aligned}\right\} \quad ---------- (4)$$

… continued

- As it is obvious that control limits will vary for each sample.
- The central line, however, will be same.
- The interpretation of these charts is similar to the *p*-chart or *d*-chart.

Application of *c*-chart

- The universal nature of Poisson distribution as the law of small numbers makes the *c*-chart technique useful.
- In spite of limited field of application of *c*-chart as compared to $\overline{X}$, *R*, *p*-chart, there do exists situations in industry where *c*-chart is definitely used.
- Some of the representative types of defects to which *c*-chart can be applied with advantage are:

1. *c* is the number of imperfections observed in a bundle of cloth.
2. *c* is the number of surface defects observed in (i) roll of coated paper or a sheet of photographic film and (ii) a galvanised sheet or a painted, plated or enamelled surface of given area.
3. *c* is the number of defects of all types observed in aircraft sub-assemblies or final assembly.
4. *c* is the number of breakdowns at weak spots in insulation in a given length of insulated wire subject to a specified test voltage.

Application of *c*-chart

5. *c* is the number of defects observed in stains or blemishes on a surface.

6. *c* is the number of soiled packages in a given consignment.

7. *c*-chart has been applied to sampling acceptance procedures based on number of defects per unit, e.g., in case of inspection of fairly complex assembled units such as T.V. sets, aircraft engines, tanks, machine-guns, etc. in which there are very opportunities for the occurrence of defects of various types and the total number of defects of all types found by inspection is recorded for each unit.

8. *c*-chart technique can be used with advantage in various fields other than industrial quality control, e.g., it has been applied

(i) to accident statistics (both of industrial accidents and highway accidents),

(ii) in chemical laboratories, and

(iii) in epidemiology.