STATISTICS

Semester IV

Unit 3.
STAT GE 404 / Statistical quality control

Topics:-

- **Importance of SQC in industrial practice**
- **Process & Product control**
- **Causes of variations in quality: chance and assignable causes**
- **Determination of tolerance limits**
- **Control charts for X-bar and R- charts.**

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STATISTICAL QUALITY CONTROL

Statistical Quality Control (SQC) refers to the use of statistical methods in the monitoring and maintaining of the quality of products and services.All the tools of SQC are helpful in evaluating the quality of services.SQC uses different tools to analyse the quality problem.Quality control is essential to building a successful business that delivers products that meet or exceed customer's expectations. It also forms the basis of an efficient business that minimizes waste and operates at high levels of productivity.It helps in maintaining the consistency of how a product is made.

The most important word in the term 'Statistical Quality Control' is quality. By quality we mean an attribute of the product that determines its fitness for use.

Dimensions of Quality:- Garvin (1987) provides an excellent discussion of eight components or dimensions of quality.

1. **Performance** (will the product do the intended job) –Potential customers usually evaluate a product to determine if it will perform certain specific functions & determine how well it performs .
2. **Reliability** (How often does the product fail) –Complex products such as many appliances,automobiles or airplanes, will require some repair over their service life.So,there are many industries in which the customer's view of quality is greatly impacted by the reliability dimensions of quality.
3. **Durability** (How long does the product last) – This is the effective service life of the product.Customers obviously want products that perform satisfactorily over a long period of time.
4. **Serviceability** (How easy is it to repair the product) – There are many industries in which the customer's view of quality is directly influenced by how quickly & economically a repair or routine maintenance activity can be accomplished.
5. **Aesthetics** (What does the product look like) –This is the visual appeal of the product ,such as style ,color,shape,packging alternative and other sensory features.
6. **Features** (What does the product do)- Usually, Customers associate high quality with products that have added features. For eg:- We might consider a spreadsheet software package to be of superior quality if it had built-in statistical analysis features while its competitors did not.

7. **Perceived Quality** (What is the reputation of the company or its product)- In many cases,Customers rely on the past reputation of the company concerning quality of its product.This reputation is directly influenced by the failures of the product that are highly visible to the public or the require product recalls. For eg :- if we you make regular business trips using a particular airline,and flight almost always arrive on time and airline company does not lose or damage your luggage, you will probably prefer to fly on that instead of its competitors.
8. **Conformance to standards** (Is the product made exactly as the designer intended) –We usually think of a high quality product as one that exactly meets the requirements placed on it. For eg:- An automobile consists of several thousand parts.If each one is just slightly too big or too small,many of the components will not fit together properly & the vehicle may not perform as the designer intended.

Few important points:-

Quality means fitness for use.

There are two general aspects of fitness for use:-

- Quality of design
- Quality of conformance

Quality of design:-All goods & services are produced in various grades or level of quality technically termed as 'quality of design' ;level or grades of quality are intentional.The design difference includes the material used in construction,specification on the component: reliability obtained through engineering development & other accessories and equipment.

Quality of conformance:- It is how well the product conforms to the specifications required by the design.Quality of conformance is influenced by the numbers of factors,including the choice of manipulating processes,the training and supervision of the workforce,the types of process controls tests and inspection activities that are employed.

Quality is inversely proportional to variability.

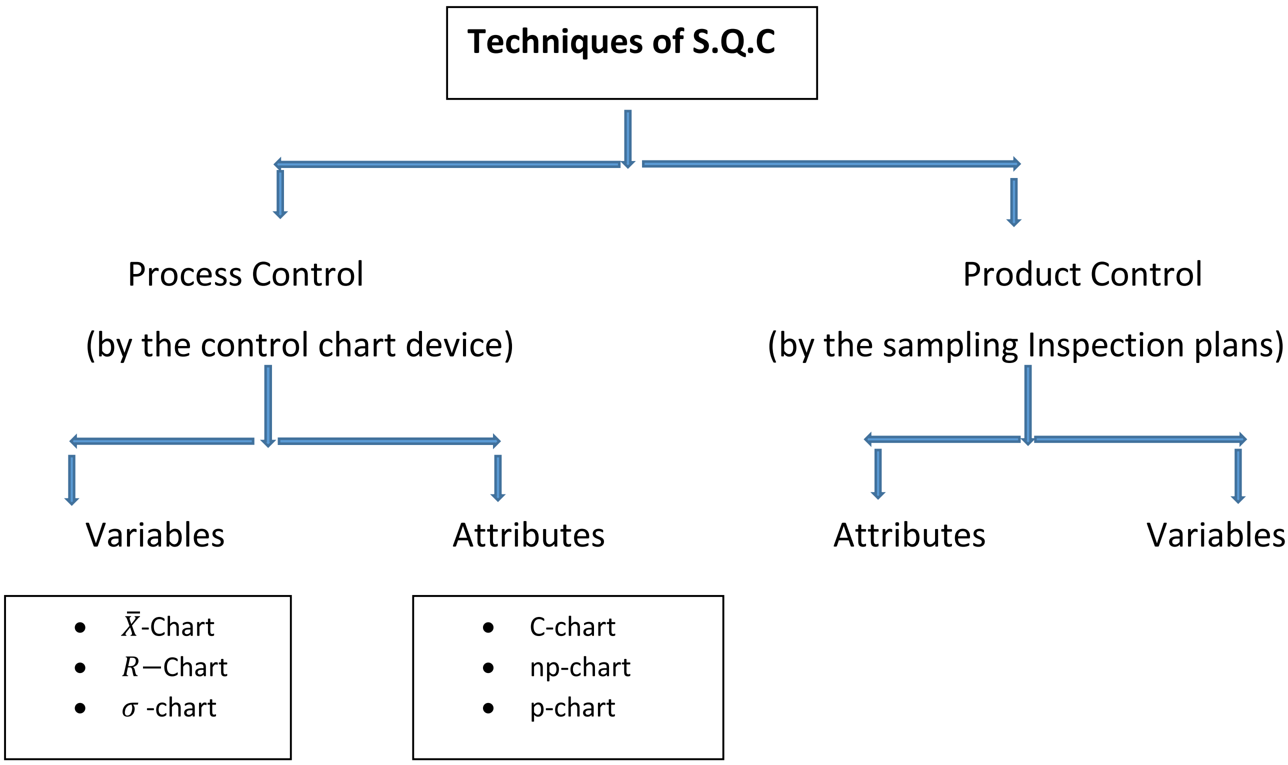
“Statistically Quality control” may be broadly defined as that industrial management technique by means of which product of uniform acceptable quality are manufactured. It is mainly concerned with setting things right rather than discovering & rejecting those made wrong – DUNCAN

PROCESS CONTROL AND PRODUCT CONTROL

The main objective in any production process is to control and maintain a satisfactory quality level of the manufactured product so that it conforms to specified quality standards.

Process Control – It is a method statistical quality control in which the quality of product is controlled while the products are in the process of production.This ensure that a satisfactory quality level of manufactured product is maintained and the proportion of defective items in the manufactured product is not too large.This is achieved through the technique of ‘control charts’ pioneered by W.A.Shewhart in 1924.

Product Control-By product control we mean controlling the quality of the product by critical examination at strategic points and the this is achieved through ‘Sampling In section plan’ pioneered by H.F.Dodge & H.C.Romig.Product control aims at guaranteeing a certain quality level is being maintained by the producer .In other words ,it attempts to ensure that the product marketed by sale department does not contain a large number of defective(unsatisfactory)items.Thus,product control is concerned with classification of raw materials ,semi-finished goods or finished goods into acceptable or rejectable itmes.



CHANCE CAUSES AND ASSIGNABLE CAUSES

The basis of statistical quality control is the degree of 'Variability' in the size or the magnitude of a given characteristic of the product.These variations are broadly classified as being due to two causes :-

i) Chance Causes

ii) Assignable Causes

Chance Causes:- In any manufacturing process,it is not possible to produce goods of exactly the same quality,variation are inevitable.Certain small variation is natural to process,being due to chance causes and cannot be prevented.The variations due to these causes is beyond the control of human hand.Thus variation is therefore called allowable variation.The range of such variation is known as 'natural tolerance of the process'.

Some typical chance causes of variation are:-

- Slight vibration of a machine.

- Lack of human perfection in reading instruments & setting controls.
- Voltage fluctuations and variation in temperature.

Assignable Causes:-This type of variation attributed to any production process is due to non-random or so called assignable causes and is termed as preventable variation.

Assignable causes may creep in at any stage of the process,right from the arrival of the raw materials to the delivery of goods.Some of the important factors of assignable causes of variation are

- Sub-standard or defective raw materials.
- Negligence of operators.
- Wrong or improper handling of machines .
- Faulty equipment.
- Unskilled or inexperienced technical staff.

  These causes can be identified and eliminated and are to be discovered in a production process before it goes wrong i.e before the production becomes defective.

  Trend pattern, shift pattern, Extreme variation pattern is used for assignable causes.

Control Limits, Specification Limits and Tolerance Limits.

1. **Control Limits**:- These are limits of sampling variation of a statistical mesure (e.g mean,range or fraction-defective) such that if production process is under control,the values of the measure calculated from different rational groups will lie within these limits. Points falling outside control limits indicate that the process is not operating under a system of chance causes i.e , assignable causes of variation are present ,which must be eliminated.Control limits are used in ‘control charts’.

2. **Specification Limits**:- When an article is proposed to be manufactured ,the manufacturers have to decide upon the maximum and minimum allowable dimensions of some quality characteristics so that the product

can be gainfully utilised for which it is is intended.If the dimensions are beyond these limits the product is treated as defective & cannot be used.These maximum and minimum limits of variations of individual items are known as “specification limits”. Specification limits are defined by customers.It is customer defined tolerance for product for service .For eg :- If manufacturer is producing a cylindrical shaft of diameter 2 cm & customers has given specification of $\pm$0.2 cm.It means a producer can produce a shaft between 1.98 cm to 2.2 cm and customer will accept.It is not a rule however it is desired that control limits should be within specification limit.So if the process goes out of the control limit still end customer does not get impacted as specification limits are outside the control limits.

3. **Tolerance Limits**- These are limits of variations of a quality measure of is expected to lie (with a given probability), provided the process is in a state of statistical quality control.For eg:- we may claim with a probability of 0.99 that at least 90% of the products will have dimensions between some stated limits.These limits are also known as ‘statistical tolerance limits’.

CONTROL CHARTS

Control chart was first proposed by Dr.Walter A Shewhart in 1924.Shewhart’s control charts provide a powerful tool of discovering and correcting the assignable causes of variation outside the ‘stable pattern’ of chance causes, thus enabling us to stabilize and control our processes & thus bring the process under statistical control.

A control chart consists of following three horizontal lines:

I. A central line (C.L) , indicating the desired standard or level of the process.
II. Upper control limit (U.C.L) ,indicating the upper limit of tolerance.
III. Lower control limit (L.C.L) ,indicating the lower limit of tolerance.

Let t be a sample statistic that measure some quality characteristic of interest.

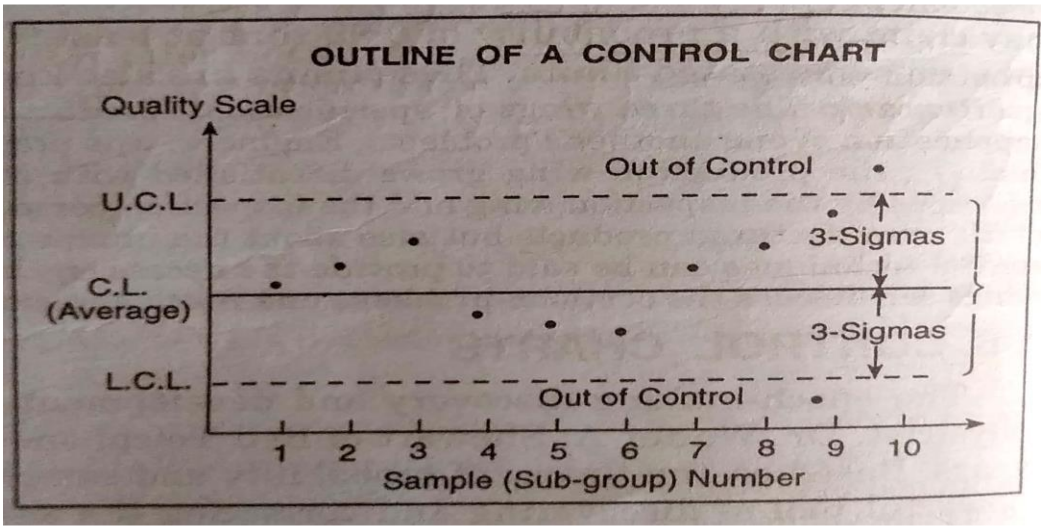
L –"distance" of the control limit from central line expresses in standard deviation unit.

i.e U.C.L= E(t) +L ∗S.E(t)   where S.E are standard error and can be presented by σ.

L.C.L= E(t) - L∗ S.E(t)

C.L = E(t)

Control charts are simple to construct &easy to interpret and tell as whether the sample point falls within 3-σ control limits or not.



CONTROL CHARTS FOR VARIABLES

1.**Shewhart's control chart for variables**:- i.e For characteristic which can be measured quantitatively. Many quality charcteristics of a product are measurable and can be expressed in specific units of measurements such as diameter of screw, life of an electric bulb,length ,thickness of steel pipe etc . Such variables are of continuous type and are regarded to follow normal probability law.

a) Charts for $\bar{X}$ (mean) and R (range)
b) Charts for $\bar{X}$ (mean) and σ (standard deviation)

Calculation of $\bar{X}$ and R for each subgroups:-

Let X_{ij} , j = 1,2,3……n be the measurements on the ith sample (I = 1,2,…k). The mean $\bar{X}_i$, the range R_i and the standard deviation S_i for the ith sample are given by

$\bar{X}_i = \frac{\sum_j X_{ij}}{n}$, R_i = max X_{ij} – min X_{ij}

$S_i^2 = \frac{\sum_j (X_{ij} - \bar{X}_i)^2}{n}$ (i= 1,2,……k)

Now, we find $\bar{\bar{X}}$ – averages of sample means.

$\bar{R}$ = sample ranges , $\bar{S}$ = sample standard deviation

$\bar{\bar{X}} = \frac{\sum_i \bar{X}_i}{k}$, $\bar{R} = \frac{\sum_i R_i}{k}$, $\bar{S} = \frac{\sum_i S_i}{k}$

- **Setting of control limits:-** σ is the process standard deviation ,then standard error of the sample mean is $\frac{\sigma}{\sqrt{n}}$.
  S.E ($\bar{X}_i$)= $\frac{\sigma}{\sqrt{n}}$

  From the sampling distribution of range ,we know that
  E(R)=$d_2.\sigma$ where d_2 is a constant depending on the sample size.Thus an estimate of σ can be obtained from $\bar{R}$.

  $\bar{R} = d_2.\sigma \quad \rightarrow \hat{\sigma} = \frac{\bar{R}}{d_2}$ …………………………(a)

  - **Control limits for $\bar{X}$- chart.**

Control Limits for $\bar{X}$-chart :

Case 1. When standards are given, i.e., both μ and σ are known. The 3-σ control limits for $\bar{X}$ chart are given by : $E(\bar{X}) \pm 3\ S.E.\ (\bar{X}) = \mu \pm 3\sigma/\sqrt{n} = \mu \pm A\sigma$, $(A = 3/\sqrt{n})$.

If μ′ and σ′ are known or specified values of μ and σ respectively, then

$$UCL_{\bar{X}} = \mu' + A\sigma' \quad \text{and} \quad LCL_{\bar{X}} = \mu' - A\sigma' \qquad \ldots(1\cdot4)$$

where $A\ (= 3/\sqrt{n})$ is a constant depending on n and its values are tabulated for different values of n from 2 to 25 in Table VIII in the Appendix.

Case 2. Standards not given. If both μ and σ are unknown, then using their estimates $\bar{\bar{X}}$ and $\hat{\sigma}$ given in Eqns. (1·2) and (1·3) respectively, we get the 3-σ control limits for the $\bar{X}$-chart as :

$$\bar{\bar{X}} \pm 3\ \frac{\bar{R}}{d_2} \cdot \frac{1}{\sqrt{n}} = \bar{\bar{X}} \pm \left(\frac{3}{d_2\sqrt{n}}\right)\bar{R} = \bar{\bar{X}} \pm A_2\bar{R}, \left(A_2 = \frac{3}{d_2\sqrt{n}}\right)$$

$$\therefore \quad UCL_{\bar{X}} = \bar{\bar{X}} + A_2\bar{R} \quad \text{and} \quad LCL_{\bar{X}} = \bar{\bar{X}} - A_2\bar{R} \qquad \ldots(1\cdot4a)$$

Since d_2 is a constant depending on n, $A_2 = 3/(d_2\sqrt{n})$ also depends only on n and its values have been computed and tabulated for different values of n from 2 to 25 and are given in the

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Control Limits for R-chart.

R-chart is constructed for controlling the variation in the dispersion (variability)of the product. The procedure of constructing R-chart is similar to that for the $\bar{X}$-chart and involves the following steps:

1.Compute the range , R_i = max X_{ij} – min X_{ij} , (i =1,2,.....,n) for each sample.

2.Compute the mean of the sample ranges:

$$\bar{R}=\frac{\sum_i R_i}{k}=\frac{(R_{1+\cdots}R_K)}{k}$$

3.Computation of control limits.- The 3-σ control limits for the R- chart are : E(R)$\pm$ 3 σ_R.

Case 1. When σ is not known

Then ,E(R) is estimated by $\bar{R}$ and σ_R is estimated from the relation.

$\sigma = d_3\hat{\sigma} = d_3.\frac{\bar{R}}{d_2}$ ....by using eq.(a)

Where d_2 and d_3 are constant depending on n.

UCL_R = E(R) + 3 σ_R = $\bar{R}$ + 3$d_3.\frac{\bar{R}}{d_2}$

UCL_R= (1+3$\frac{d_3}{d_2}$) $\bar{R}$ = $D_4.\bar{R}$

LCL_R = E(R) - 3 σ_R = $\bar{R}$ + 3$d_3.\frac{\bar{R}}{d_2}$ = (1-3$\frac{d_3}{d_2}$) $\bar{R}$ = $D_3.\bar{R}$

The values of D_4 and D_3 depend only on n.

Case 2. When σ is known ,then

UCL_R = E(R) + 3 σ_R = $d_2.\sigma$ + 3 $d_3.\sigma$ =(d_2 + 3 d_3)σ =$D_2\sigma$

LCL_R = E(R) + 3 σ_R = $d_2.\sigma$ - 3 $d_3.\sigma$ =(d_2 - 3 d_3)σ =$D_1\sigma$

CL_R=E(R) =$\bar{R}$

Since range can never be negative , LCL_R must be greater than or equal to 0. In case it comes out to be negative ,it is taken as zero.
