O. C. Curve of Double Sampling Plan

Semester IV

STAT CC410

Unit 3

- **O. C. Curve of Double Sampling Plan**
- **Consumer's and Producer's Risk**
- **A. S. N. and A. T. I. of Double Sampling Plan**

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O. C. Curve of Double Sampling Plan

The lot will be accepted under the following mutually exclusive ways:

(i) $0 \le d_1 \le c_1$

(ii) $d_1 = c_1 + 1,\ d_2 \le c_2 - d_1 \Rightarrow d_2 \le c_2 - c_1 - 1$

(iii) $d_1 = c_1 + 2,\ d_2 \le c_2 - d_1 \Rightarrow d_2 \le c_2 - c_1 - 2$

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(.) $d_1 = c_2, \quad d_2 = 0$

Hence, by addition theorem of probability, the probability of acceptance for the lot of incoming quality '*p*' is given by,

$$P_a(p) = \sum_{x=0}^{c_1} g(x,p) + \sum_{y=0}^{c_2-x} \sum_{x=c_1+1}^{c_2} g(x,p).h(y,p \mid x)$$

where $g(x, p)$ is a probability of finding x defectives in the first sample and $h(y, p \mid x)$ is the conditional probability of finding y defectives in the second sample under the condition that x defectives have already appeared in the first sample. Thus,

$$g(x,p) = \frac{\binom{Np}{x}\binom{N-Np}{n_1-x}}{\binom{N}{n_1}}$$

$$h(y,p \mid x) = \frac{\binom{Np-x}{y}\binom{N-n_1-(Np-x)}{n_2-y}}{\binom{N-n_1}{n_2}}$$

Hence, we get,

$$P_a(p) = \sum_{x=0}^{c_1} \frac{\binom{Np}{x}\binom{N-Np}{n_1-x}}{\binom{N}{n_1}} + \sum_{y=0}^{c_2-x}\sum_{x=c_1+1}^{c_2} \frac{\binom{Np}{x}\binom{N-Np}{n_1-x}\binom{Np-x}{y}\binom{N-n_1-Np+x}{n_2-y}}{\binom{N}{n_1}\binom{N-n_1}{n_2}} \quad \text{…..... (1)}$$

=> $P_a(p) = P_{a_1}(p) + P_{a_2}(p)$ (say)

where $P_{a_1}(p)$ and $P_{a_2}(p)$ are the probabilities of acceptance on the basis of first and second samples respectively.

Consumer's and Producer's Risk

The consumer's risk is given by,

P_c = P [Accepting a lot of quality p_t] = $P_a(p_t)$ ……………. (2)

and producer's risk is given by,

P_p = P [Rejecting a lot of quality $\bar{p}$]

= 1 - P [Accepting a lot quality $\bar{p}$]

= 1 - $P_a(\bar{p})$ ……………. (3)

Hence, replacing 'p' by p_t and $\bar{p}$ in (1) and substituting in (2) and (3), we get consumer's risk and producer's risk respectively.

A. S. N. and A. T. I. of Double Sampling Plan

In an acceptance-rejection double sampling plan, the number of items inspected for a lot is either n_1 (when the lot is accepted or rejected on the basis of the first sample) or $(n_1 + n_2)$ when a second sample of size n_2 is drawn, Thus, the expected sample size for a decision is given by,

$$\begin{aligned} ASN &= n_1 P_1 + (n_1 + n_2)(1 - P_1) \\ &= n_1 + n_2(1 - P_1) \end{aligned}$$

where P_1 is the probability of a decision (acceptance or rejection of the lot) on the basis of the first sample.

However, in a double sample acceptance-rejection scheme in which rejected lots are inspected 100% the average total inspection (ATI) per lot is given by,

$$ATI = n_1 P_{a_1} + (n_1 + n_2) P_{a_2} + N(1 - P_a) \quad \text{……………. (4)}$$

Since

(i) only n_1 items will be inspected if the lot is accepted on the basis of the first sample and its probability is $P_{a_1}(p)$,

(ii) $(n_1 + n_2)$ items will be inspected if the lot is accepted on the basis of the second sample and its probability is $P_{a_2}(p)$, and

(iii) the entire lot of N items will be inspected if the lot is rejected and the probability of this is $[1 - P_a(p)]$.

Since, $P_a = P_{a_1} + P_{a_2}$ => $P_{a_2} = P_a - P_{a_1}$

Then from (4) we get,

$$\begin{aligned} ATI &= n_1 P_{a_1} + (n_1 + n_2)(P_a - P_{a_1}) + N(1 - P_a) \\ &= n_1 P_{a_1} + (n_1 + n_2)\left[(1 - P_{a_1}) - (1 - P_a)\right] + N(1 - P_a) \\ &= n_1 + n_2(1 - P_{a_1}) + (N - n_1 - n_2)(1 - P_a) \qquad \ldots\ldots\ldots\ldots\ldots (5) \end{aligned}$$

Note: - In Dodge and Romig tables, n_2 has no fixed relation to n_1 but is determined so that *ATI* is minimum and so that the probability of acceptance on the basis of first sample is approximately the same as the probability of acceptance on the basis of second sample.