Determination of n and c

Semester IV

STAT CC410

Unit 3

- **Determination of *n* and *c***
- **Binomial Approximation to Hyper-geometric Distribution**

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Determination of *n* and *c*

The lot size N is usually known. Thus, the two unknown quantities need to be determined in the sampling plan are n and c.

In a lot of quality p, the number of defective pieces is Np and non-defective pieces $N - Np = N(1-p)$. The probability of getting exactly x defectives in a sample of size n from this lot is getting by Hyper-geometric distribution as,

$$g(x,p) = \frac{\binom{Np}{x}\binom{N-Np}{n-x}}{\binom{N}{n}}$$

Probability of accepting a lot of quality p is

$$P_a(p) = \sum_{x=0}^{c} g(x,p) = \sum_{x=0}^{c} \frac{\binom{Np}{x}\binom{N-Np}{n-x}}{\binom{N}{n}} \qquad \ldots\ldots\ldots\ldots\ldots (1)$$

Hence, the consumer's risk is given by,

P_c = P [Accepting a lot of quality p_i]

$$= \sum_{x=0}^{c} g(x,p_i) = \sum_{x=0}^{c} \frac{\binom{Np_i}{x}\binom{N-Np_i}{n-x}}{\binom{N}{n}} \qquad \ldots\ldots\ldots\ldots\ldots (2)$$

To protect himself against poor quality, the consumer usually demands a small value of P_c for given p_i.

The producer's risk is given by,

P_p = P [Rejecting a lot of quality $\bar{p}$]

= 1 - P [Accepting a lot of quality $\bar{p}$]

$$= 1 - \sum_{x=0}^{c} g(x,\bar{p}) = 1 - \sum_{x=0}^{c} \frac{\binom{N\bar{p}}{x}\binom{N-N\bar{p}}{n-x}}{\binom{N}{n}} \qquad \ldots\ldots\ldots\ldots\ldots (3)$$

If process average fraction defective is $\bar{p}$ as claimed by the producer then the average amount of total inspection per lot is

$$ATI = n + (N-n)P_p \qquad \ldots\ldots\ldots\ldots\ldots (4)$$

since n items have to be inspected in each case and the remaining (N – n) items will be inspected only if $d > c$.

The computation of hyper-geometric probabilities in (2) and (3) is very difficult and in practice the binomial approximation to hyper-geometric distribution is used.

Thus, a convenient and practical substitute for (2) becomes:

$$P_c = \sum_{x=0}^{c} \frac{(Np_i)!}{x!(Np_i - x)!} \left(\frac{n}{N}\right)^x \left(1 - \frac{n}{N}\right)^{Np_i - x} \qquad \ldots\ldots\ldots\ldots\ldots (5)$$

and the substitute of (3) becomes,

$$P_p = 1 - \sum_{x=0}^{c} \frac{n!}{x!(n-x)!} (\bar{p})^x (1 - \bar{p})^{n-x} \qquad \ldots\ldots\ldots\ldots\ldots (6)$$

In most of the practical problem $\bar{p}$ is likely to be less than 0.10 and n is likely to be sufficiently large to warrant the use of Poisson approximation to binomial distribution. Thus, (6) can further be approximated by,

$$P_p^{\,*} = 1 - \sum_{x=0}^{c} \frac{(n\bar{p})^x e^{-n\bar{p}}}{x!} \qquad \ldots\ldots\ldots\ldots\ldots (7)$$

And consequently,

$$A.T.I. = n + (N - n)\left[1 - \sum_{x=0}^{c} \frac{(n\bar{p})^x e^{-n\bar{p}}}{x!}\right] \qquad \ldots\ldots\ldots\ldots\ldots (8)$$

Consumer’s requirement fixes the values of P_c and p_i. N is always fixed. For given values of P_c and p_i the equation (2) which involves two unknowns n and c is satisfied by a large number of pairs of n and c. To safeguard producer’s interest also, out of these possible pairs one involving the minimum amount of inspection as given in (4) is chosen. Though theoretical computations are quite cumbersome and time consuming, Dodge and Romig, by applying numerical methods of solution of equations, have prepared extensive tables for minimizing values of n and c for $P_c = 0.10$ and different values of $\bar{p}$.

Note:- Binomial Approximation to Hyper-geometric Distribution

Consider the hyper-geometric distribution with probability function

$$p(x) = \frac{\binom{m}{x}\binom{N-m}{n-x}}{\binom{N}{n}} \qquad \text{................ (i)}$$

If N and n are large and m and (N – m) is relatively small, the hyper-geometric distribution (i) can be approximated by binomial distribution with parameters m and $p = n/N$. Under these conditions, we get,

$$\frac{\binom{m}{x}\binom{N-m}{n-x}}{\binom{N}{n}} \to \binom{m}{x}\left(\frac{n}{N}\right)^{x}\left(1-\frac{n}{N}\right)^{m-x} \qquad \text{................ (ii)}$$

If N, m and (N – m) are large relative n and x, then (i) can be approximated by binomial distribution B (n, $p = n/N$).

Thus,

$$\frac{\binom{m}{x}\binom{N-m}{n-x}}{\binom{N}{n}} \to \binom{n}{x}\left(\frac{m}{N}\right)^{x}\left(1-\frac{m}{N}\right)^{n-x} \qquad \text{................ (iii)}$$