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INTRODUCTION TO LOGIC

Logic is the science of reasoning. It is the study of the methods and principles used to distinguish between correct and incorrect reasoning. Aristotle was the great logician. His treaties on logic later combined into one great work entitled as '**The Organon' (**The Instrument**)** which constitutes the earliest formal study of our subject. Formal logic deals with the concepts of truth and validity. It forms the categorical syllogism with the help of laws of thoughts.

Logic is further divided into various types

1. Formal Logic
2. Informal Logic
3. Symbolic Logic
4. Deductive Logic
5. Inductive Logic

SYMBOLIC LOGIC: USE OF SYMBOLS

Symbolic logic deals with the symbols in order to solve the logical arguments. Symbolic logic is the method of representing logical expressions through the use of symbols and variables, rather than in ordinary language. This has the benefit of removing the ambiguity that normally accompanied in ordinary languages, such as English and also allow easier operation.

IMPORTANCE OF SYMBOLIC LOGIC

1. The use of symbols in symbolic logic helps us to bring out the features of logical importance in arguments and classify them into types – The traditional method of classifying arguments into types which was first invented by Aristotle involves the use of symbols. The use of variables in logic enables us to state general rules for testing the validity of arguments. Thus one important function of symbols in logic is to express the generality of the rules of logic.

2. Another important use of symbols in logic is to give conciseness and economy of expression to complicated statements which would be difficult or impossible to understand if they were expressed in ordinary language.
3. Special symbols in logic representing logical operations facilitated the appraisal of arguments.
4. It can also be used to prove basic theorems using truth tables. But most importantly, learning logic teaches you how to think. It teaches you what is incorrect reasoning, to recognize fallacies, to check soundness of arguments, etc.
5. Symbolic logic is by far the simplest kind of logic – it is a great time saver in argumentation. Additionally, it helps prevent logical confusion.

PROPOSITIONAL LOGIC

SIMPLE AND COMPOUND PROPOSITIONS -

Propositional Logic deals with arguments containing simple and compound statements. The two types of statements dealt within propositional logic are, simple and compound statements. The modern logicians have broadly classified propositions into simple and compound propositions.

A simple statement is one that does not contain any other statement as a component. A simple proposition cannot be analysed into further propositions. For example, ‘Ramesh is honest’. A simple statement cannot be further analysed into statement or statements. ‘Ramesh is honest’ does not contain any other statement as a component.

A compound statement is one that does contain another statement as a component. For example, ‘Ramesh is not honest’. ‘Ramesh and Dinesh are honest’. A compound statement can be further analysed into a statement or statements. ‘Ramesh is not honest’ can be analysed into ’Ramesh is honest’ which is a statement and ‘not’ which is a word. It contains ‘Ramesh is honest’ as a component. ‘Ramesh and Dinesh are honest ‘can be further analysed into ‘Ramesh is honest’ which is a statement and ‘Dinesh is honest’ which is another statement. It contains ‘Ramesh is honest’ as a component and ‘Dinesh is honest’ as another component.

There are five types of compound propositions.

They are, Negation, Conjunction, Disjunction, Conditional statement and Bi-Conditional statement.

TRUTH-FUNCTIONAL COMPOUND STATEMENTS:

Negation ; Conjunction, Disjunction; Conditional Statement and Material Implication.

The notion of a function may be made clear with the help of n example from mathematics. Y=X+4 is an expression in mathematics. The value taken by X decides the value of Y. If X is 2, then Y must be 6. Here Y is a function of X, because the value of X decides the value of Y. In a similar way in the expression Z=3X-3Y+X, Z is said to be the function of X and Y. In logic, instead of the numerical values, truth-values namely true and false are used. The truth-value of a true proposition is “true” and the truth-value of a false proposition is “false”.

A compound proposition is truth-functional if and only if it’s truth-value is completely determined by the truth-values of its component statements. There are several types of truth-functional compound statements.

Conjunction - Conjunction is a compound proposition in which the word “and” is used to connect simple statements. Conjunction of two statements is formed by placing the word “and” between them. The two component statements of conjunction are called “conjuncts”. For example, ‘Ramesh is honest and Dinesh is intelligent’. ‘Ramesh is honest’ is the first conjunct and ‘Dinesh is intelligent’ is the second conjunct. To have a unique symbol whose only function is to connect statements conjunctively, the dot “.” symbol is used for conjunction. The above conjunction is represented as “Ramesh is honest. Dinesh is intelligent”. It is symbolized as” R.D” where’ R’ represents “Ramesh is honest, ‘D’ represents ‘Dinesh is intelligent”. More generally, where p and q are any two statements whatever their conjunction is symbolized as ‘p . q’.

In conjunction, if both it’s conjuncts are true , the conjunction is true, otherwise it is false. For this reason a conjunction is said to be a truth- functional compound statement. The dot “ . ” symbol is a truth-functional connective.

Given any two statements, p and q, there are four possible sets of truth- values they can have, which can be displayed as follows:

Where p is true and q is true, p.q is true.

Where p is true and q is false, p.q is false.

Where p is false and q is true, p.q is false.

Where p is false and q is false, p.q is false.

If we represent the truth-values “true” and “false” by the capital letters T and F, the truth-table for conjunction can be represented as follows:

P	q	p.q
T	T	T
T	F	F
F	T	F
F	F	F

As shown by the truth-table defining the dot symbol, a conjunction is true if and only if both of it's conjuncts are true.

Negation - Negation is a compound proposition in which the word 'not" or the phrase 'it is not the case that' or the phrase "It is false hat"is used. Example,'Ramesh is not honest' or 'It is not the case that Ramesh is honest' or 'It is false that Ramesh is honest'. It contains 'Ramesh is honest' as component part.

Negation of a statement is formed by the insertion of the word "not" in the original statement or by prefixing to it the phrase 'it is not the case that' or 'it is false that'. The symbol " ~ " called "curl" or "tilde" is used to form the negation of a statement. Thus the above example is represented as ~R where R symbolizes 'Ramesh is honest'. More generally, where p is any statement whatever, it's negation is symbolized as ' ~ p'.

Negation is a truth-functional compound statement and the curl " ~ " is a truth-functional operator. The negation of any true statement is false, and the negation of any false statement is true. The truth- table for Negation can be represented as follows:

P	~ P
T	F
F	T

Disjunction - Disjunction is a compound proposition in which the simple propositions are connected by th word 'or ' or the phrase 'either……….or'. Disjunction or alternation of two statements is formed by inserting the word "or" or the phrase "either….or" between them. The two component statements are called "disjuncts" or " alternatives". For example,' A or B', 'Either A or B'. The symbol "v" is a truth-functional connective. The four truth-value possibilities are ,

where "p" is true and "q" is true, "pvq" is true.

Where" p" is true and "q" is false,"pvq" is true.

Where "p" is false and "q" is true, "pvq" is true.

Where "p" is false and "q" is false, "pvq" is false.

The truth-table for disjunction is as follows:

P	q	pvq
T	T	T

T	F	T
F	T	T
F	F	F

<u>Implication or Conditional Statement</u> - Implication is a compound proposition in which the simple statements are connected by the phrase ' if …… then'. For example, "If it rains, then the road will be wet". The part of proposition which lies in between 'if' and 'then' is called the antecedent or implicans. The part of proposition which follows the word 'then' is called the consequent or implicate . The general form of an implicative proposition is as follows: "If antecedent, then consequent".

The symbol " Ɔ " can be regarded as representing another kind of implication called "material implication". The possible combinations of truth-values for implication is represented as follows:

Where p is true and q is true, p Ɔ q is true.

Where p is true and q is false, p Ɔ q is false.

Where p is false and q is true, p Ɔ q is true.

Where p is false and q is false, p Ɔ q is true.

The truth-table for implication is as follows:

P	q	p Ɔ q
T	T	T
T	F	F
F	T	T
F	F	T

<u>Bi-conditional proposition</u> - Bi-conditional proposition is a compound proposition in which the simple statements are connected by the phrase 'if and only if'. For example, "I will go to the cinema if and only if my friend comes with me". Bi-conditional proposition is also called 'material equivalence'. When two statements are combined by using the phrase "if and only if", the resulting compound statement is called bi-conditional statement or Material Equivalence. The phrase "If and only if" is symbolized as ′ ≡ '.

Where p and q are any two statements whatever, their material equivalence is represented as 'P ≡ q '. When two statements are materialy equivalent, they materially imply each other. The possible combinations of truth-values for material equivalence is as follows:

Where p is true and q is true, p ≡ q is true .

Where p is true and q is false, p ≡ q is false.

Where p is false and q is true, p ≡ q is false.

Where p is false and q is false , p ≡ q is true.

Two statements are said to be materially equivalent when they have the same truth-value. The symbol ′ ≡ ‘ is a truth-functional connective. The truthtable for biconditional or material equivalence is as follows:

P	q	p ≡ q
T	T	T
T	F	F
F	T	F
F	F	T