PATNA WOMEN'S COLLEGE

Course: MCA **Semester**: II
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Unit 3: Numerical Integration

WEDDLE'S RULE

We have the general quadrature formula for numerical integration as follows:

$$I = h\left[ny_0 + \frac{n^2}{2}\Delta y_0 + \left(\frac{n^3}{3} - \frac{n^2}{2}\right)\frac{\Delta^2 y_0}{2!} + \left(\frac{n^4}{4} - n^3 + n^2\right)\frac{\Delta^3 y_0}{3!} + \ldots upto(n+1) terms\right] \quad \ldots.(i)$$

Putting n=6 in the formula (i) and neglecting all differences of seventh and higher order, we get

$$\int_{x_0}^{x_0+6h} ydx = h\left[6y_0 + 18\Delta y_0 + 27\Delta^2 y_0 + 24\Delta^3 y_0 + \frac{123}{10}\Delta^4 y_0 + \frac{33}{10}\Delta^5 y_0 + \frac{41}{140}\Delta^6 y_0\right]$$

Here, the coefficients of $\Delta^6 y_0$ differs from $\frac{3}{10}$ by the small fraction $\frac{1}{140}$. Hence if we replace this coefficient by $\frac{3}{10}$, we commit an error of only $\frac{h}{140}\Delta^6 y_0$. If the value of h is such that the sixth differences are small, the error committed will be negligible. We, therefore, change the last term to $\left(\frac{3}{10}\right)\Delta^6 y_0$ and replace all differences by their values in terms of the given y's. The result becomes,

$$\int_{x_0}^{x_0+6h} ydx = \frac{3h}{10}\left[y_0 + 5y_1 + y_2 + 6y_3 + y_4 + 5y_5 + y_6\right]$$

Similarly,

$$\int_{x_0+6h}^{x_0+12h} ydx = \frac{3h}{10}\left[y_6 + 5y_7 + y_8 + 6y_9 + y_{10} + 5y_{11} + y_{12}\right]$$

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$$\int_{x_0+(n-6)h}^{x_0+nh} y\,dx = \frac{3h}{10}\left[y_{n-6} + 5y_{n-5} + y_{n-4} + 6y_{n-3} + y_{n-2} + 5y_{n-1} + y_n\right]$$

[If n is a multiple of 6]

Adding all these integrals, we have, if n is a multiple of 6

$$\int_{x_0}^{x_0+nh} y\,dx = \frac{3h}{10}\left[y_0 + 5y_1 + y_2 + 6y_3 + y_4 + 5y_5 + 2y_6 + 5y_7 + y_8 + \ldots\ldots\ldots\right]$$

This formula is known as Weddle's Rule.

It is more accurate, in general, than Simpson's rule, but it requires at least seven consecutive values of the function.

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Example: Calculate the value of the given integral by Weddle's rule

$$\int_4^{5.2} \log x dx$$

Solution: Taking

$$h = \frac{5.2 - 4}{6} = 0.2$$

Divide the whole range of integration (4, 5.2) into six equal parts. The values of the logx for each point of sub-division are given below:

x	logx
x_0=4.0	y_0=1.3862944
x_0+h=4.2	y_1=1.4350845
x_0+2h=4.4	y_2=1.4816045
x_0+3h=4.6	y_3=1.5260563
x_0+4h=4.8	y_4=1.5686159
x_0+5h=5.0	y_5=1.6094379
x_0+6h=5.2	y_6=1.6486586

By Weddle's rule, we have

$$\int_4^{5.2} \log x dx = \frac{3h}{10}\left[y_0 + y_6 + 5(y_1 + y_5) + y_2 + y_4 + 6y_3\right]$$

$$= \frac{3(.2)}{10}[1.3862944 + 1.6486586 + 5(1.4350845 + 1.6094379) + 1.4816045 + 1.56866159 + 6(1.5260563)]$$

$$= \frac{.6}{10}[3.034953 + 15.222612 + 3.0502204 + 9.1653378]$$

$$= \frac{.6}{10}[30.464123] = 1.8278474 \text{ Ans.}$$

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