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MCA CS2T07: *Automata Theory*

Pushdown Automata (Finite Automata + Stack)

Pushdown Automata is a ***finite* automata with stack**. It is acceptor of *Context Free Grammar*. It could read the letters of an input string perform stack operations and make state changes.

The execution of a PDA starts with one **symbol** (Z_0 or as per our design) on the stack. PDA uses ***three*** stack operations as below:

1. POP: The pop operation reads top symbol and removes it from the stack.
2. PUSH: This operation writes a symbol onto the top of the stack.
3. NOP: It does nothing to the stack.

PDA is a **seven** tuples machine.

$$PDA = \{Q, \Sigma, \delta, q_0, Z_{0,} F, \Gamma\}$$

Where,

Q	is the finite set of states,
Σ (uppercase/capital sigma)	*input symbol,*
δ	is the *next move function* or transition
	Deterministic PDA => $Q\ X\ \{\sum U\ \in\}\ X\ \Gamma \rightarrow Q\ X\ \Gamma^*$ $(one\ state)$
	Non-Deterministic PDA=>$Q\ X\ \{\sum U\ \in\}\ X\ \Gamma \rightarrow 2^{(Q\ X\ \Gamma^*)}$ $(more\ than\ one\ state)$
q_0	in Q is the *start state*,
Z_0	Bottom of the stack
F ⊆ Q	is the set of *final states*
Γ	stack *symbols*

Types of PDA

Deterministic: The PDA is deterministic when at most one move is possible from any ID (Instantaneous Descriptions) i.e., a PDA $M = \{Q, \Sigma, \delta, q_0, Z_0, F, \Gamma\}$ is deterministic if:

1. for each q in Q and Z in Γ, ehenever $\delta(q, \in, Z)$ is nonempty then $\delta(q, a, Z)$ is empty for all a in Σ; (***This condition prevents the possibility of a choice between a move independent of the input symbol (**$\in -\boldsymbol{move}$**) and move involving an input symbol***)
2. for no q in Q, Z in Γ and a in Σ ∪ {∈} does $\delta(q, a, Z)$ contain more than one element. (***This condition prevents a choice of move for any***$(\boldsymbol{q}, \boldsymbol{a}, \boldsymbol{Z})$***) or*** $\boldsymbol{\delta}(\boldsymbol{q}, \in, \boldsymbol{Z})$).

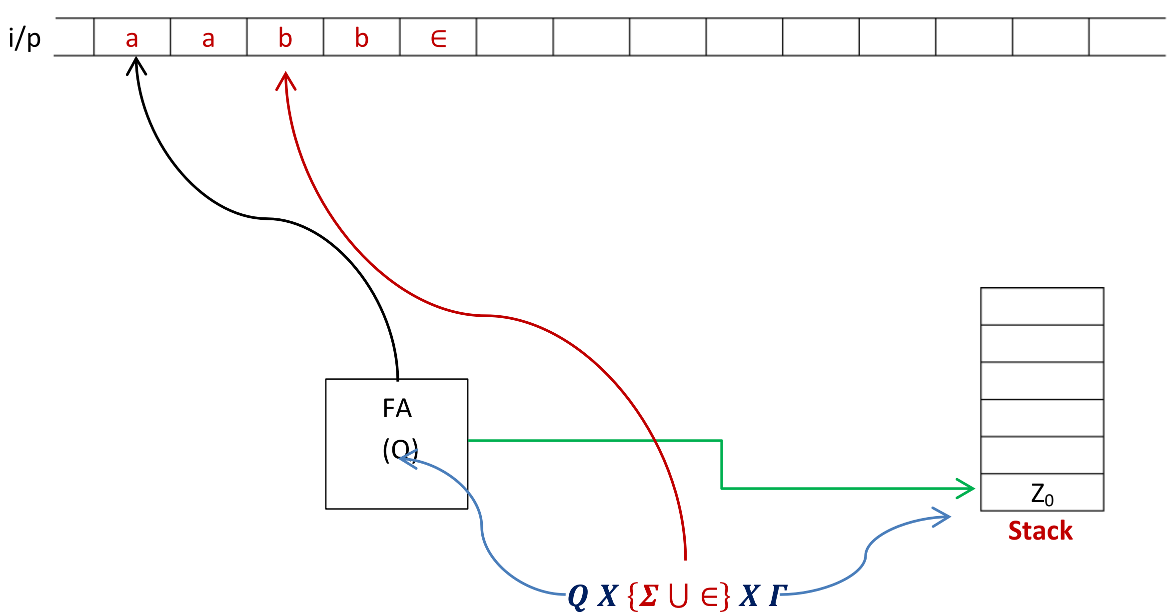
Example: $\boldsymbol{L} = \{\boldsymbol{WcW^R} | \boldsymbol{w} \in (\boldsymbol{0} + \boldsymbol{1})^*\}$

Non-Deterministic: A non-deterministic PDA M accepts an input if any sequence of choices causes M to empty its stack. Thus M always "guesses right" because wrong guesses, in themselves, do not cause an input to be rejected. An input is rejected only if there is no "right guess".

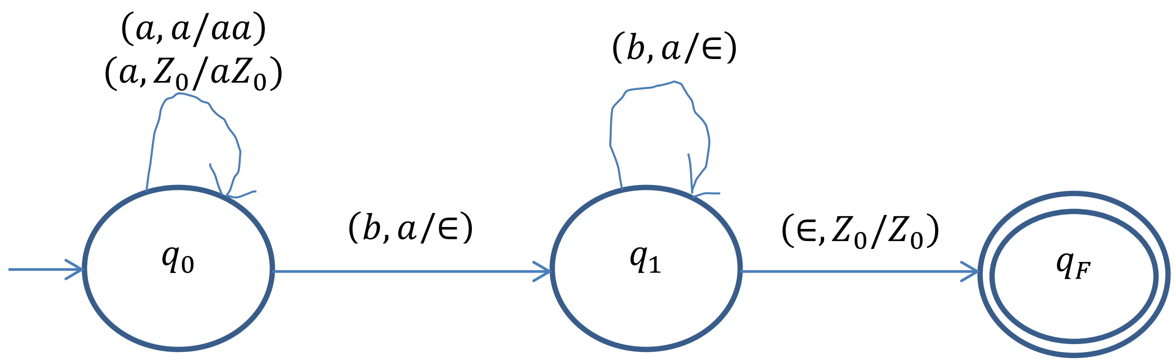
Example: $\boldsymbol{L} = \{\boldsymbol{WW^R} | \boldsymbol{w} \in (\boldsymbol{0} + \boldsymbol{1})^*\}$

Deterministic PDA:

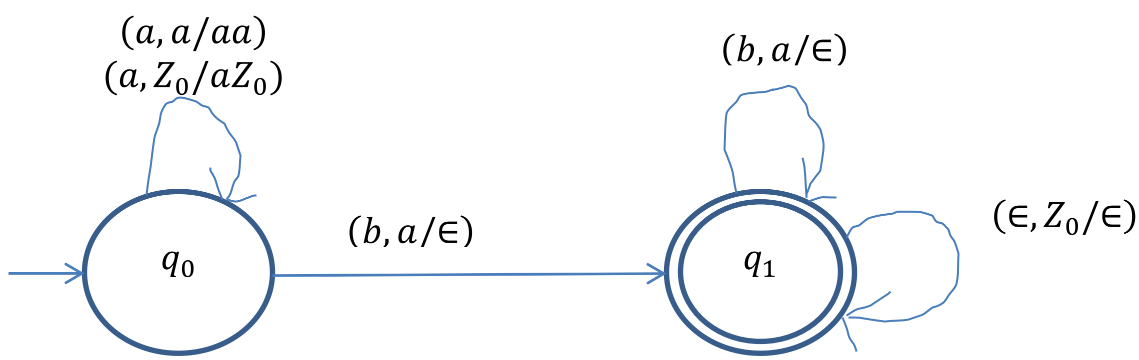
Example 1: ***Design a deterministic PDA of*** $\boldsymbol{L} = \{\boldsymbol{a^n b^n} | \boldsymbol{n} \geq \boldsymbol{1}\}$.



Here, w = aabb∈



Acceptance by final State



Acceptance by empty stack

Transition:

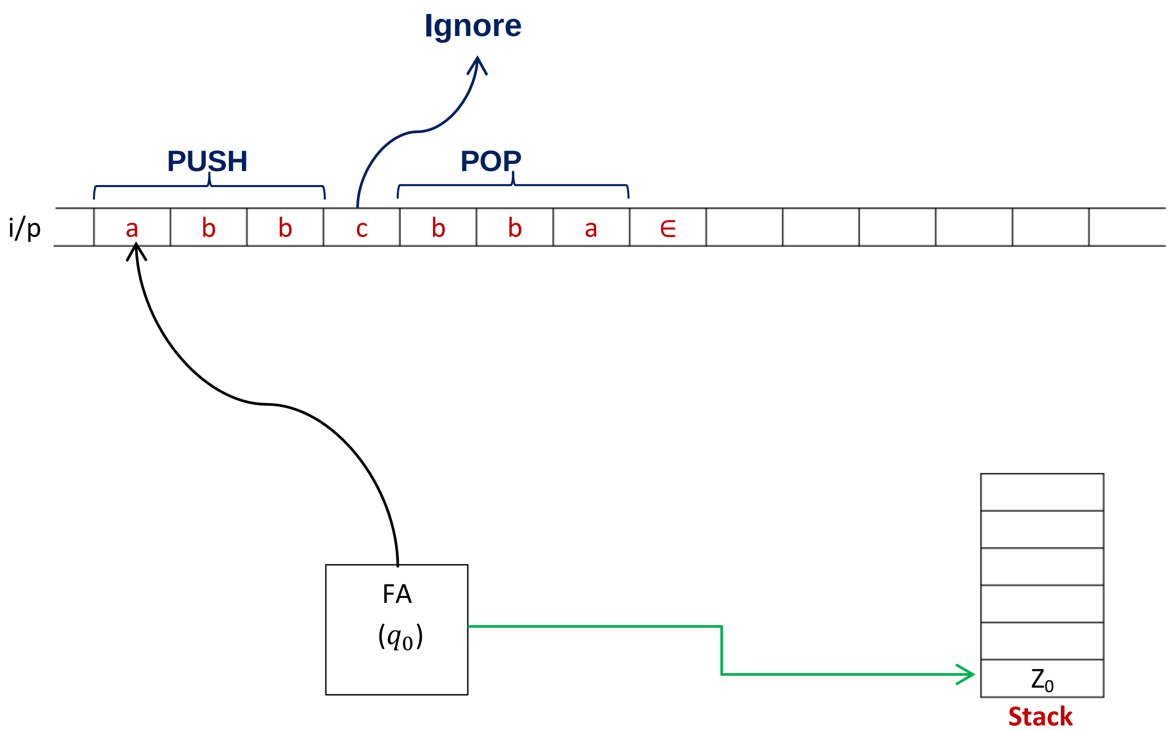
$\delta(q_{0,}, a, Z_0) = (q_{0,}, aZ_0)$
$\delta(q_{0,}, a, a) = (q_{0,}, aa)$
$\delta(q_{0,}, b, a) = (q_{1,}, \in)$
$\delta(q_{1,}, b, a) = (q_{1,}, \in)$
$\delta(q_{1,}, \in, Z_0) = (q_{F,}, aZ_0)$ or $(q_{1,}, \in)$

Example 2: ***Design a deterministic PDA of*** $\boldsymbol{L = \{WcW^R | w \in (a+b)^*\}}$.

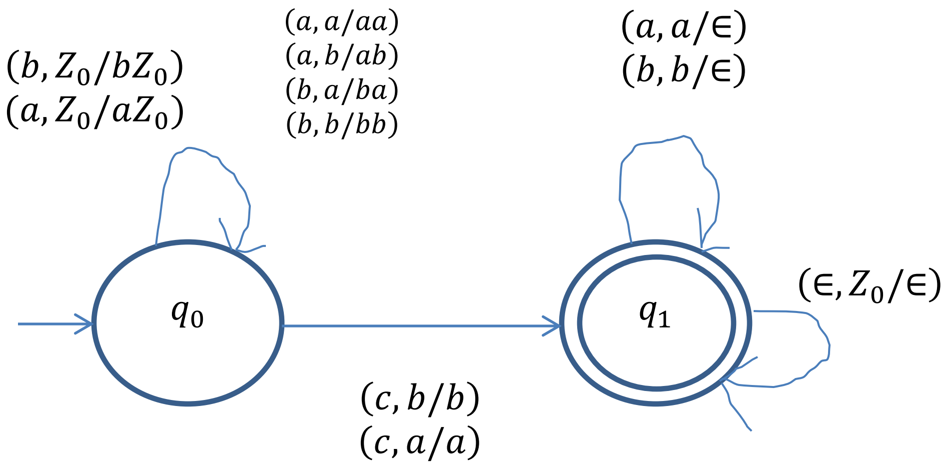
Solution:

w={$\in, a, b, aa, ab, bb, aba, bba, \ldots., aabaab, \ldots \ldots$..}

Let we take **w=abb** then **w^R=bba** therefore **wcw^R=abbcbba**



Here, w = abbcbba∈



Transition diagram for $\boldsymbol{L} = \{\boldsymbol{WcW^R} | \boldsymbol{w} \in (\boldsymbol{a} + \boldsymbol{b})^*\}$

Transition Function:

Initial condition: $\delta(q_{0,}, a, Z_0) = (q_{0,}, aZ_0)$

$\delta(q_{0,}, b, Z_0) = (q_{0,}, ab)$

PUSH: $\delta(q_{0,}, a, a) = (q_{0,}, aa)$

$\delta(q_{0,}, b, a) = (q_{0,}, ba)$

$\delta(q_{0,}, a, b) = (q_{0,}, ab)$

$\delta(q_{0,}, b, b) = (q_{0,}, bb)$

Ignore: $\delta(q_{0,}, c, a) = (q_{1,}, a)$

$\delta(q_{0,}, c, b) = (q_{1,}, b)$

POP: $\delta(q_{1,}, a, a) = (q_{1,}, \in)$

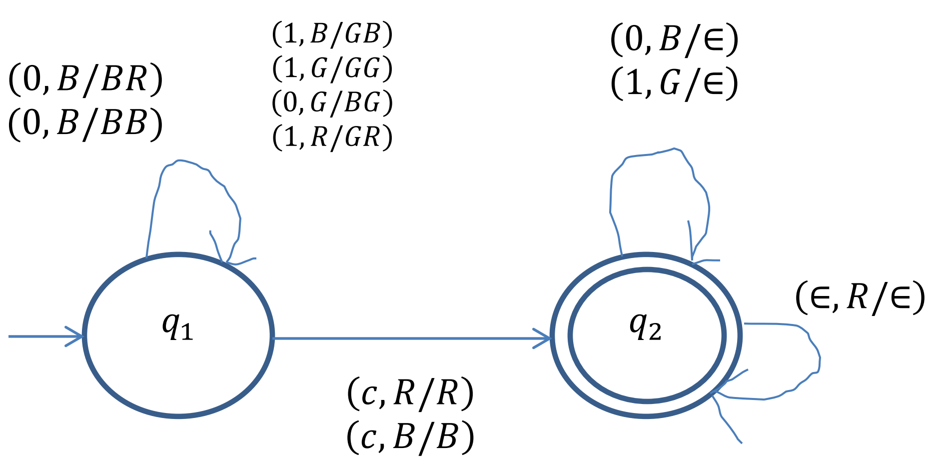
$\delta(q_{1,}, b, b) = (q_{1,}, \in)$

Final: $\delta(q_{1,}, \in, Z_0) = (q_{1,}, \in)$

Example 3: ***Consider the following transitions:***

$$\delta(q_{1,}, 0, R) = (q_{1,}, BR)$$
$$\delta(q_{1,}, 0, B) = (q_{1,}, BB)$$
$$\delta(q_{1,}, 0, G) = (q_{1,}, BG)$$
$$\delta(q_{1,}, 1, R) = (q_{1,}, GR)$$
$$\delta(q_{1,}, 1, B) = (q_{1,}, GB)$$
$$\delta(q_{1,}, 1, G) = (q_{1,}, GG)$$
$$\delta(q_{1,}, c, R) = (q_{2,}, R)$$
$$\delta(q_{1,}, c, B) = (q_{2,}, B)$$
$$\delta(q_{1,}, c, G) = (q_{2,}, G)$$
$$\delta(q_{2,}, 0, B) = (q_{2,}, \in)$$
$$\delta(q_{2,}, \in, R) = (q_{2,}, \in)$$
$$\delta(q_{2,}, 1, G) = (q_{2,}, \in)$$

a. Write the grammar for above transition. => **CFG**
b. Which automaton represents the above transition? => **Pushdown Automaton**
c. Name the type of automaton. => **Deterministic Push Down Automata**
d. Show the language of above transition. => $\boldsymbol{L} = \{\boldsymbol{WcW^R} | \boldsymbol{w} \in (\mathbf{0} + \mathbf{1})^*\}$
e. Show the pictorial diagram of above transition.



Transition diagram for $\boldsymbol{L} = \{\boldsymbol{WcW^R} | \boldsymbol{w} \in (\mathbf{0} + \mathbf{1})^*\}$