Qubits are realized as actual physical systems but can also be treated as mathematical objects. Bits work on pure states |0> and |1>.These notations | > is known as Dirac notation.

Quantum Gates & Quantum Circuits:

To operate an information processing task, we need a series of single-qubits gates, which need perform operation on 1 qubit at time. (Barenco et al., 1995) Then we need two qubits gate. With these two gates, we can implement any quantum logic circuits i.e., required to solve the computational probe Bell States. It is a circuit which has a HADAMARD gate followed by CNOT gate, Transform four computational basis states. First Hadamard transform puts top qubits in a superposition then act as control input to the CNOT, and target gets inverted only when the control is 1. The output states are known as bell states.

$$|\beta_{00}\rangle = \frac{(|00\rangle + |11\rangle)}{\sqrt{2}}$$

$$|\beta_{01}\rangle = \frac{(|01\rangle + |10\rangle)}{\sqrt{2}}$$

$$|\beta_{10}\rangle = \frac{(|00\rangle - |11\rangle)}{\sqrt{2}}$$

$$|\beta_{11}\rangle = \frac{(|01\rangle - |10\rangle)}{\sqrt{2}}$$

The General decomposition of two-qubits quantum gates:

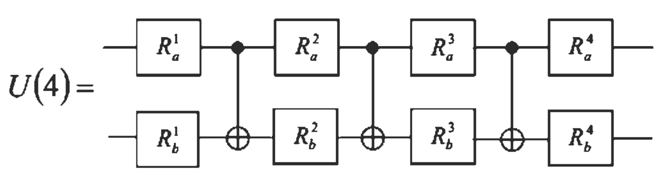

Fig. 1 – Two-qubit gate constructed using eight single-qubit gates and three CNOTs.

A general two-qubit gate is parameterized by fifteen variables (three for the non-local operation and twelve for the local's operations). Using (3) -(4), two-qubit gate can be constructed using three CNOT gates and eight single-qubit gates (Vidal and Dawson 2003).

A universal quantum circuit for three-qubit quantum gate:

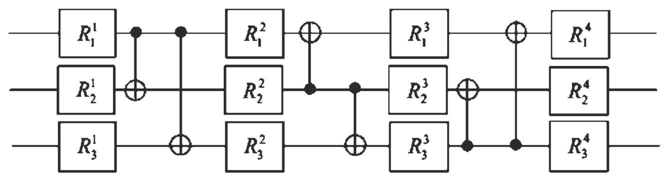

Fig. 2 – Universal cell for three-qubits quantum gates.

The universal quantum circuit provides the possibility of implementation of a quantum version of the FPGA (Field Programmable Gate Array) technology, which permits the hardware being set via software. In fact, assembling some basic cells for an n-qubit quantum circuit, as shown in Fig. 2, several different quantum operations can be implemented just adjusting the parameters of the single-qubit gates and enabling or not the CNOT gates (Peres, 1985). When a CNOT gate is not enabled, it will work as an identity gate.

Entanglement Theory:

- A laser (usually ultraviolet for its high frequency) sends a photon through a nonlinear crystal such as Beta Barium Borate.
- The photon bumps an electron to an excited state When the electron comes back down and releases its photon, there is a chance it will split
- If it splits, the two photons are equally half of the energy.
- These two photons are entangled

- The overlapping of the cones represents the entanglement.
- The two photons are also polarized opposite of one another.

Superposition:

One of the properties that sets a qubit apart from a classical bit is that it can be in superposition. Superposition is one of the fundamental principles of quantum mechanics (Vidal and Dawson 2003). In classical physics, a wave describing a musical tone can be seen as several waves with different frequencies that are added together, superposed. Similarly, a quantum state in superposition can be seen as a linear combination of other distinct quantum states. This quantum state in superposition forms a new valid quantum state.

Quantum superposition is fundamentally different from superposing classical waves. A quantum computer consisting of n qubits can exist in a superposition of 2^n states: from $|000...0\rangle$ to $|111...1\rangle$. In contrast, playing n musical sounds with all different frequencies, can only give a superposition of n frequencies. Adding classical waves scales is linear, where the superposition of quantum states is exponential.

i) There are the most entangled states possible.

ii) The most entangled state means that if one takes a particle trace of the systems density matrix over the states of the qubit B in order to obtain the density operator for the qubit A then one will get

$\rho_a = \mathrm{Tr}\ \beta\,(|\beta_{ij}> <\beta_{ij}|) = 1/2\ I_A$

iii) The latter means that the result of one measurements is absolutely random. A qubit in the state l0> with probability of 1\2 or in a state l1> with same probability after the measurement. Thus, we will not obtain any information about the prepared state after a local measurement of A or B qubits.

iv) From bell state $\{|\beta_{ij}|\}$ from an orthogonal basis for a two qubit Hilbert space. The measurement of the qubit in the Bell state should be proceeded in the appropriate basis.

v) The measurement of the qubit in the Bell state.

Conclusion:

In the present theoretical study, we learnt about the quantum computer basics (Grover 1998) The unit of the information in quantum computation known as qubit has also been studies & its different representation. There is a closed resemblance between the classical logic gates & the quantum qubit gates. In the quantum gates, the input is quantum bits, abbreviated as qubit.

We have discussed about the Shor's Algorithm using which the single qubits gates can be generalized to form higher order gates. Among the single qubits gates we have Hadamard, CNOT and other single input gates. The higher order two and three qubit gates can be formed from single qubit CNOT gates and Hadamard gates (Lukac 2009). Also, we got to find out that the logical operation are results of unitary transformation involving Paul Spin matrices & Identity matrix.

Acknowledgement:

We would like to express our gratitude to our Principal, Dr. Sister M. Rashmi A.C. who provided us this golden opportunity to do this project under BSR scheme.

References:

Barenco A, Bennett C H, Cleve R, DiVinchenzo D P, Margolus N, Shor P, Sleator T, Smolin J A, and Weinfurter H, (1995). Elementary Gates for quantum computation. *The American Physical Society,* 52: 3457–3467.

Grover L K (1998). Quantum computers can search rapidly by using almostany transformation. *Phys. Rev. Lett.,* 80 (19) : 4329–4332.

Lukac M (2009). Quantum logic synthesis and inductive machine learning, Ph.D. dissertation, *Portland State University.* pp. 8-10

Nielsen Michael A and Chuang Isaac L (2000). *Quantum Computation and Quantum Information. Cambridge University Press.* pp. 17-22.

Peres A, (1985). Reversible logic and quantum computers. *Phys. Rev. A,* 32 (6) : 3266– 3276.

Pittman T B, Fitch M J, Jacobs B C, and Franson J D (2003). Experimental controlled-NOT logic for single photons in the coincidence basis, *Phys. Rev. A,* 68: 032316/14.

Spector L, Barnum H, Bernstein H J, and Swamy N (1999). Quantum computing applications of genetic programming, *Advan. in Gen. Programming,* 3:4-9

Vidal G and Dawson C M (2003). A universal quantum circuit for two-qubit transformations with three CNOTs gates. *quant- ph,* 0307177.