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Mathematical Modeling of Predator–Prey Systems

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Abstract: *This study explores the dynamics of predator–prey interactions using the classical Lotka–Volterra model. The model is used to understand how two species—one acting as prey and the other as predator—affect each other's population over time. Through numerical analysis, changes in population sizes are observed under different parameter conditions. The results show a repeating pattern where prey populations increase first, followed by a rise in predator populations. It is also observed that changes in growth rate or predation intensity can significantly impact system stability, sometimes even leading to extinction. Overall, the study highlights how sensitive ecological systems are to parameter variations and shows the importance of mathematical models in analyzing such relationships.*

Keywords: *Mathematical modelling, Lotka-volterra model, Equilibrium points, Population stability, Phase plain analysis*

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Introduction:

Mathematical modeling plays an important role in understanding real-world systems by representing them through equations. In ecology, it helps explain how different species interact and how their populations change over time. One of the most well-known models in this field is the predator–prey model, which describes the relationship between two species where one depends on the other for survival.

The Lotka–Volterra model, developed independently by Alfred J. Lotka and Vito Volterra in the early 20th century, provides a simple yet powerful way to study these interactions. It shows that predator and prey populations often follow a cyclic pattern rather than reaching a fixed value. In this work, we focus on analyzing this model, identifying its equilibrium points, and understanding how different parameters such as growth rate and death rate influence the system. This helps in gaining a clearer idea of how balance is maintained in natural ecosystems.

Objectives:

1. To model the interaction between prey and predator species using mathematical equations.
2. To investigate how various factors such as birth rates, death rates, and interaction rates influence population stability.
3. To identify the thresholds at which predator or prey populations may collapse or thrive.

Related Literature:

Lotka (1920) and Volterra (1926) were pioneers in developing mathematical models that describe the cyclical relationship between predator and prey populations. Holling (1959) introduced the concept of the functional response, which describes how the rate of predation depends on prey density. Rosenzweig and MacArthur (1963) introduced carrying capacity for prey, modifying the Lotka–Volterra equations to include environmental limitations. Murray (2002) emphasized parameter sensitivity in biological systems, showing that small changes in growth or death rates can alter population stability. Edelstein-Keshet (2005) expanded nonlinear dynamics to study more complex ecological systems involving multiple interacting species. Recent studies have also applied numerical simulation techniques such as the Runge–Kutta method to analyze system stability and phase plane trajectories. These models continue to serve as a foundation for ecological management and population control strategies.

Methodology:

The Lotka–Volterra model consists of a system of first-order nonlinear differential equations that describe the rate of change in prey and predator populations over time.

Model Equations:

$dx/dt = \alpha x - \beta xy$

$dy/dt = -\gamma y + \delta xy$

where:

- α — growth rate of prey
- β — predation rate coefficient
- γ — natural death rate of predator
- δ — rate of predator increase due to predation

Assumptions:

1. In the absence of predators, prey population grows exponentially at rate α.
2. In the absence of prey, predator population declines exponentially at rate γ.
3. The rate of predation is proportional to the product of predator and prey populations.

Analysis and Equilibrium Points:

The system reaches equilibrium when both population derivatives are zero: $dx/dt = 0$ and $dy/dt = 0$. This gives two equilibrium points: (0, 0) — extinction of both species (theoretical case), and $(x^*, y^*) = (\delta/\beta, \alpha/\gamma)$ — coexistence equilibrium.

Threshold Analysis:

- Prey threshold: $x^* = \gamma/\delta$ — minimum prey population for predator survival.
- Predator threshold: $y^* = \alpha/\beta$ — maximum predator population that prey can sustain.

Parameter Effects:

- Higher α: more prey growth, leading to increased predator population.
- Higher γ: predator decline, allowing prey to multiply.
- Higher β or δ: more intense predator-prey interaction, increasing oscillation amplitude.

Graphical Representation:

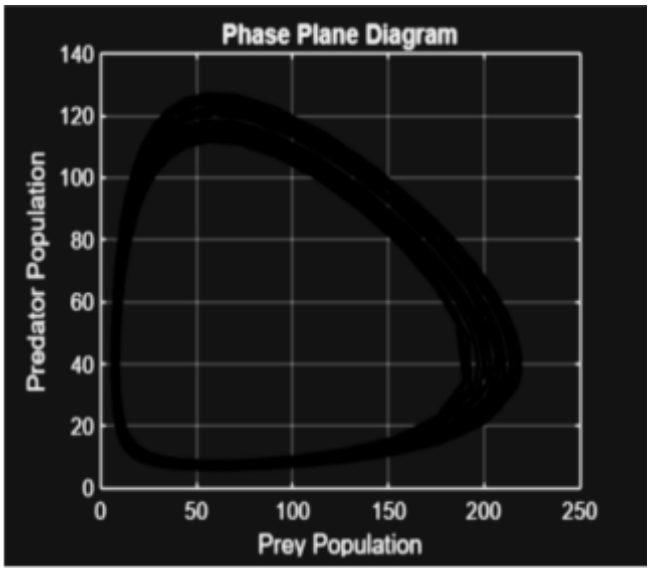


Fig. 1. Population oscillation of prey and predator over time

Conclusion:

The Lotka–Volterra model serves as a useful tool for understanding the interaction between predator and prey populations. From the analysis, it is clear that the behavior of the system depends strongly on parameters such as growth rate, death rate, and interaction coefficients. Even small changes in these values can lead to noticeable differences in population trends. The model also demonstrates that predator and prey populations tend to follow a cyclic pattern rather than

remaining constant. Although the model is simplified and does not include all real-world factors, it still provides valuable insights into ecological balance. This makes mathematical modeling an important approach for studying and predicting ecosystem behavior.

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