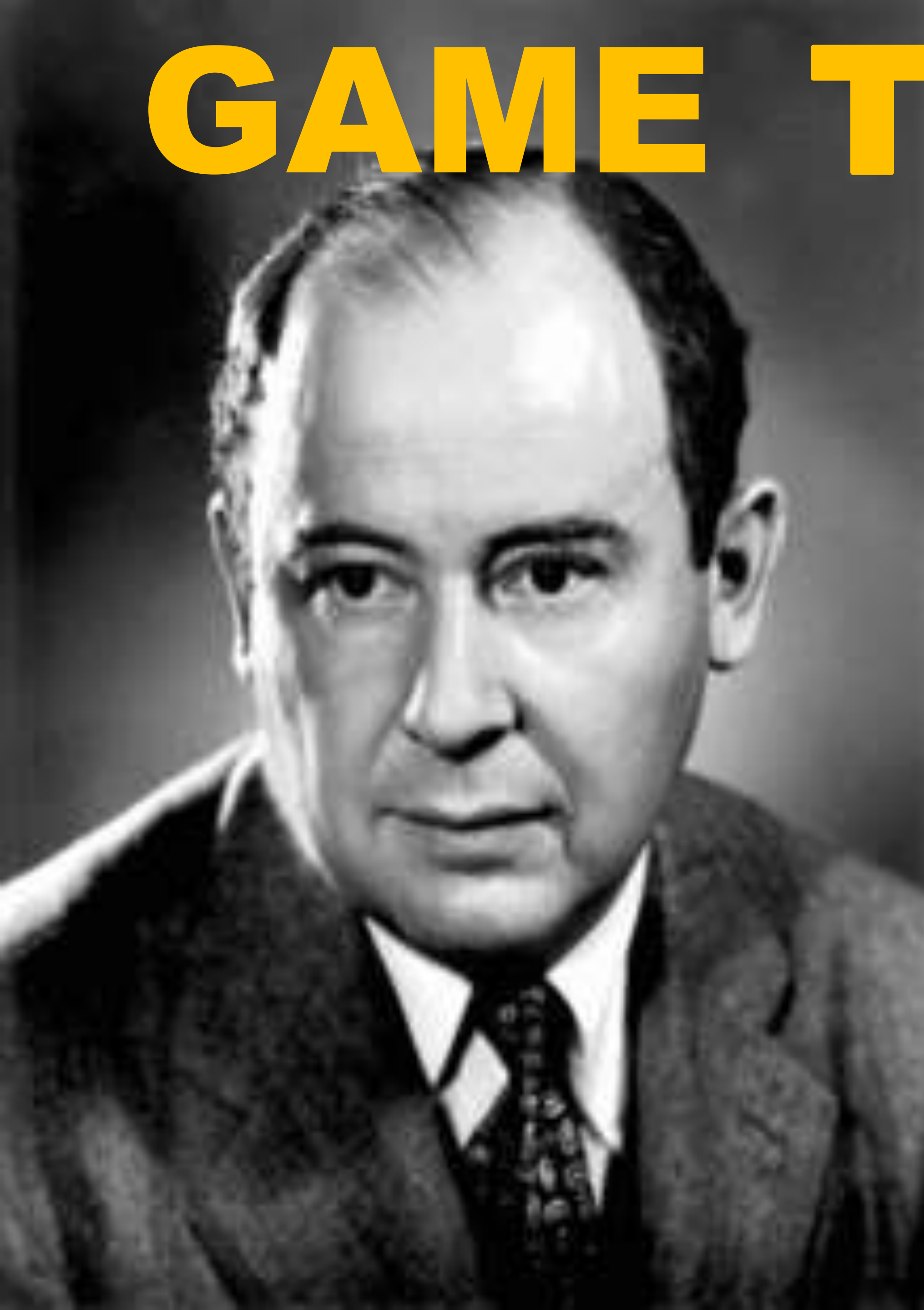
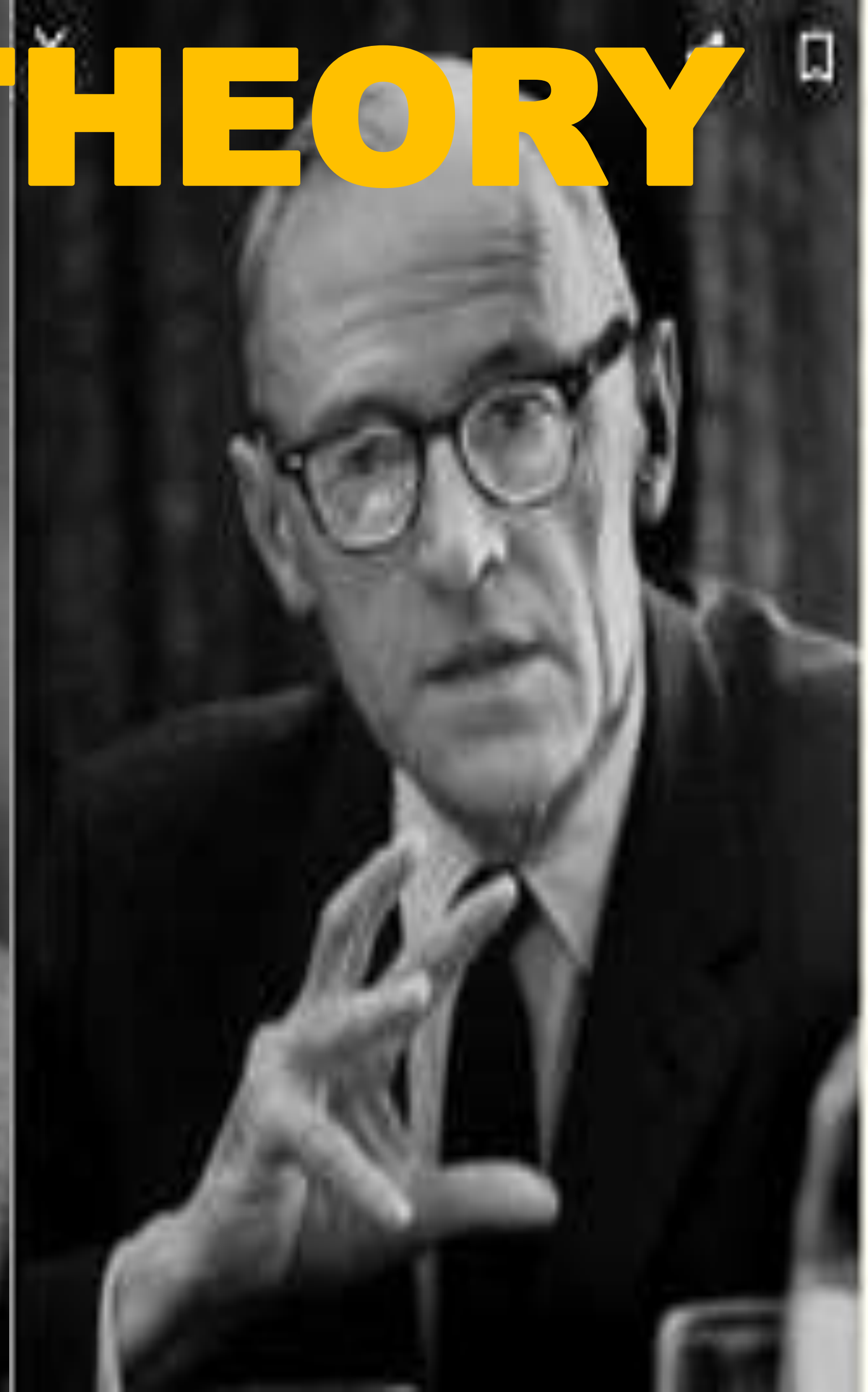
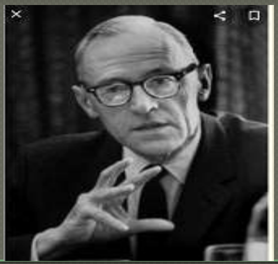
GAME THEORY





HISTORY:-



- The History of Game Theory goes back to the early part of 20th Century with publication in1838 of Augustin Cournot"Researches into Mathematical Principles of the Theory of Wealth".

- Momentum has gained with works of Emile Borel(1921) & By John Von Neumann andOskar Morgenstern with publication of the book"THEORY OF THE GAMES AND ECONOMIC BEHAVIOUR".

- Some of the other Contributors are Nobel Laureates John Nash,John Harsanyi and Reinhard Selten.

.

INTRODUCTION

- Game theory is basically a study of conflicts & competition involving individuals, firms or any other combination .
- The concepts are utilized to formulate, analyze and study the strategic behaviour.
- Assumptions are-(1)Players are Rational.
  (2)Course of action mostly driven by the one which maximises gain at the cost of its opponent.

KEY TERMINOLOGIES

Players
(decision-makers)

Pay-Off(outcome of the game- win , lose or draw)

STRATEGY

Pure Strategy
(select same strategy)

Mixed Strategy
(uses combination of strategies)

Value of the Game
(Expected pay-off of a game – optimum strategies)

Pay- Off Matrix
Gains or losses from a game represented in a matrix form.

Types of Game:-

1. Two-Person
2. Multiperson
3. Two-Person Zero sum
4. Two-Person non-zero sum

Saddle Point
Maximin of one player=Minimax of another player

Dominance Rule
Some strategies are superior and acc. To the principle , size of pay-off matrix can be reduced by deleting those strategies which are dominated by others.

PAY OFF MATRIX OF TWO PERSON ZERO SUM GAME

PLAYER II

PLAYER I $\begin{pmatrix} 4 & 6 \\ 7 & 8 \end{pmatrix}$

Given above is **the pay off matrix of a zero sum two person game**.We have to find the best strategy for each player and the value of the game.

Solution:- Player I plays the first strategy (i.e Row I) his gain is either 4 or 6 depending on the strategy followed by Player II.Here least outcome will be 4 units for him.

Similarly, if Player I adopts Row 2 as his strategy, then minimum gain is 7units whatever be the course of action (i.e selection of column) by second player.

	Player II		Row Minimum	
	4	6	4	
Player I	7	8	7	← Maximin
Column Maximum	7	8		
	↑ Minimax			

Aim of Player I is to maximise gain it will be natural tendency of Player II to minimize his maximum loss i.e selection between 7 and 8 units.Therefore,7 units represents saddle point and also the value of the game i.e gain of units to Player I.

DOMINANCE RULE

- Some strategies stands superior or dominates the other i.e the outcome is always preferred overall other in all conditions.
- According to the principle of Dominance ,in such cases the size of the pay off matrix can be reduced by deleting those strategies which are dominated by others. Probability of selecting dominated strategies is 0.
- Similar situation exist for opponent where each element in a particular column stand superior(dominating column) than corresponding elements of another column(dominated column).Eliminate dominating column.

NASH EQUILIBRIUM

- It is named after an American Mathematician , John Nash.
- It is a decision making theorem within game theory that states a player can achieve the desired outcome by not deviating from their initial strategy.
- In Nash Equilibrium ,each player' s strategy is optimal when considering the decisions of other players. Every player wins because everyone gets the outcome they desire.
- The Prisoners Dilemma is a common example and one that adequately showcases the effect of the Nash Equilibrium.
- It is the most generic method such that dominance rule or successive dominant method become a part of it.
- The players have no tendency to deviate thus it is the most stable strategy.

	Advertisement	**No Advertisement**
Advertisement	A:10	A:15
	B:5	B:0
No Advertisement	A:6	A:20
	B:8	B:2

- In case where firm A has no dominant strategy ,the equilibrium state is reached where both firms adopt advertising as the best choice. Therefore ,they have an incentive to change their strategies.

PRISONER'S DILEMMA

❖An important game model that has significant implications for the behaviour of oligopolists is popularly known as Prisoner' s Dilemma .

❖It explains how rival firms behave selfishly act contrary to their mutual or common interest.

▪Two accomplice were arrested for committing a robbery.

▪The two of them under the law could not be arrested due to lack of evidence between them.

▪Each suspect were interrogated in isolation so that there is no exchange of communication between them.

▪The Attorney or Public Prosecutor gave an offer that there will be no punishment for the one who confesses and severe punishment for the other party.

▪Now, if each accomplice do not confess, then both can escape punishment and if both confess, they will get imprisonment of say 5 years. Thus, each suspect has 2 strategies open to them either to confess or not to confess:-

		B' s Strategy	
	No Confession	No Confession	confession
A's Strategy		A:0 B:0	A:10 B:0
	Confession	A:0 B:10	A:5 B:5

MIXEDSTRATEGIES(Solution of a 2X2 game without saddle point)

In order to determine formulation of such cases , we determine probabilities with which each course of action is chosen. So, we consider a game whose pay-off matrix is of the form:-

player B

player A $\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$

If Player A chooses his i^{th} course of action with probabilities p^i and player B choose j^{th} course of action with probability q^j

$$.p^1 = \frac{a22-a21}{(a11-a12)+(a22-a21)}, \quad p^2 = \frac{a^{11}-a^{12}}{(a11-a12)+(aa22-a21)}, \quad q^1 = \frac{a22-a12}{(a11-a12)+(a22-a21)}, \quad q^2 = \frac{a^{11}-a^{21}}{(a11-a12)+(a22-a21)}$$

Thus, $$V = \frac{a^{11}a^{22}-a^{12}a^{21}}{(a^{11}-a^{12})+(a^{22}-a^{21})}$$

ARITHMETIC METHOD

It is used in obtaining optimal strategies for each player in (2X2) games without saddle points. The steps involved are:

1. Find the difference of 2 numbers in Row I and put it under the Row II neglecting the neg. Sign if occurs.
2. Find the difference of 2 numbers in Row II and put it under the Row I , neglecting the neg. Sign if occurs.
3. Repeat the same for 2 columns.

The Value obtained by swapping the difference represents the optimal relative frequencies of both the players . These are the frequencies can be converted to probabilities by dividing each of them by their sum.

For eg. Player Y

Player X(10 11) 12-8=4,p1=4/5

12 8 11-10=1,p2=1/5

q1=3/5,11-8=3 14-12=2,q2=2/5

MATRIX METHOD

If, pay-off matrix of a game takes a form of sq. matrix, then optimal solution can be obtained by matrix method.

The solution of a two person Zero sum game with mixed strategies with a square pay-off matrix can be calculated as

$$\text{Player A's Optimal strategy} = \frac{[1 \quad 1]P_{adj}}{[1 \quad 1]P_{adj}\begin{bmatrix}1\\1\end{bmatrix}}$$

$$\text{Player B's Optimal Strategy} = \frac{[1 \quad 1]P_{cof}}{[1 \quad 1]P_{adj}\begin{bmatrix}1\\1\end{bmatrix}}$$

Therefore, the value of the game is given by –

V=[Player A's optimal strategies]X[Player B's optimal strategies]X[Pay-off matrix P_{ij}]

Algebraic Method

Consider a game with pay-off matrix of the form$[a_{ij}]mXn$.

Let ,V be thevalue of the game and $(p_1,p_2,p_3.....p_m)$ and $(q_1,q_2,q_3.....q_n)$ are the respective probability of selection of optimal strategies by the players A & B.

The expected gain to be recievedby Player A when Player B uses any of his strategies from 1st,2nd , nth strategies are

$\sum_{i=1}^{m} a_{i1}pi, \sum_{i=1}^{m} a_{i2}pi, \sum_{i=1}^{m} a_{i3}pi,...... \sum_{i=1}^{m} a_{m}pi$ respectively.

Since, the player A is the gainer and expects atleast 'V'

Therefore, $\sum_{i=1}^{m} a_{1j}pi \geq v$, j=1,2,3....n;where, $\sum_{i=1}^{m} pi=1$

Similarly,by considering the losses to B when Player A uses the respective strategies we get the following system of inequalities.

$\sum_{j=1}^{n} a_{1j}q_{j,} \sum_{j=1}^{n} a_{2j}q_j, \sum_{j=1}^{n} a_{3j}q_j,...... \sum_{j=1}^{n} a_{mj}q_j$

Player B is the loser.So,

$\sum_{j=1}^{n} a_{1j}q_j \leq v$,where, $\sum_{j=1}^{n} qj=1$.

To get the value of the game,we solve the inequalities.

THANK YOU !